Mixing beliefs among interacting agents

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ABSTRACT. *We present a model of opinion dynamics where agents adjust continuous opinions on the occasion of random binary encounters whenever their difference in opinion is below a given threshold. High thresholds yield convergence of opinions towards an average opinion, but low thresholds result in several opinion clusters: members of the same cluster share the same opinion but do not adjust any more with members of other clusters.*

1. Introduction

Most models about opinion dynamics [FOL 74], [ART 94], [ORL 95], [LAT 97], [GAL 97], [WEI 99], are based on binary opinions which social actors update under either social influence or according to their own experience. One issue of interest concerns the importance of the binary assumption: what would happen if opinion were a continuous variable such as the worthiness of a choice (a utility in economics), or some belief about adjustment of a control parameter? In some European countries, the Right/Left political choices were often considered as continuous and were represented for instance by the geometrical position of the seat of a deputy in the Chamber.

Binary opinion dynamics under imitation processes have been well studied, and we expect that in most cases the attractor of the dynamics will display uniformity of opinions, either 0 or 1, when interactions occur across the whole population. This is the "herd" behaviour often described by economists [FOL 74], [ART 94], [ORL 95].

1. Laboratoire associé au CNRS (URA 1306), à l'ENS et aux Universités Paris 6 et Paris 7

Clusters of opposite opinions appear when the dynamics occurs on a social network with exchanges restricted to connected agents. Clustering is reinforced when agents diversity is introduced, for instance diversity of influence [LAT 97], [GAL 97], [WEI 99].

The a priori guess for continuous opinions is also homogenisation, but towards average initial opinion [LAS 89]. The purpose of this paper is to present results about continuous opinion dynamics when convergent opinion adjustments only proceed when opinion difference is below a given threshold. We will give results concerning homogeneous mixing across the whole population and mixing across a social network. Preliminary results about binary vectors of opinions will also be presented.

2. Complete Mixing

2.1. *The basic model*

Let us consider a population of N agents i with continuous opinions x_i. At each time step any two randomly chosen agents meet. They re-adjust their opinion when their difference of opinion is smaller in magnitude than a threshold d. Suppose that the two agents have opinion x and x' and that $|x - x'| < d$; opinions are then adjusted according to:

$$x = x + \mu \cdot (x' - x)$$
$$x' = x' + \mu \cdot (x - x')$$

Where μ is the convergence parameter taken between 0 and 0.5 during the simulations.

The rationale for the threshold condition is that agents only interact when their opinion are already close enough; otherwise they do not even bother to discuss. The reason for such behaviour might be for instance lack of understanding, conflicts of interest or social pressure. Although there is no reason to suppose that openness to discussion, here represented by threshold d, is constant across population or even symmetrical on the occasion of a binary encounter, we will allways take it as a constant simulation parameter in the present paper.

The evolution of opinions can be mathematically predicted in the limit case of small values of d [NEA 00]. Density variations $\delta\rho(x)$ of opinions x obeys the following dynamics:

$$\delta\rho(x) = \frac{d^3}{2} \cdot \mu \cdot (\mu - 1) \cdot \frac{\partial^2(\rho^2)}{\partial x^2}$$

This implies that starting from an initial distribution of opinions in the population, any local higher opinion density is amplified. Peaks of opinions increase and valleys are depleted until very narrow peaks remains among a desert of intermediate opinions.

2.2. *Results*

Figures 1 and 2 obtained by computer simulations, display the time evolution of opinions among a population of $N = 1000$ agents for two values of the threshold d. Initially opinions were randomly generated across a uniform distribution on [0,1]. At each time step a random pair is chosen and agents re-adjust their opinion according to equation 1 and 2 when their opinions are closer than d. Convergence of opinions is observed, but uniformity is only achieved for the larger value of d.

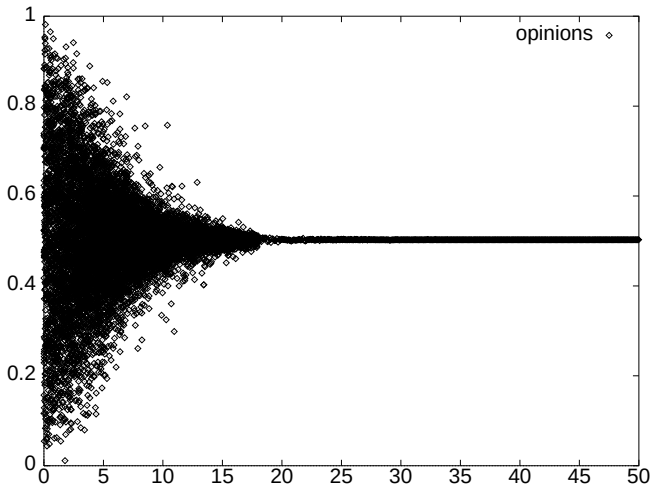

Figure 1. *Time chart of opinions ($d = 0.5 \quad \mu = 0.5 \quad N = 2000$). One time unit corresponds to sampling 1000 pairs of agents.*

Another way to follow agents opinion dynamics is to plot final opinions as a function of initial opinions. The plot on figure 3 shows how final opinions "reflect" initial opinions for $\mu = 0.5$: some overlap of initial opinions are seen in clusters frontiers. The overlap observed when $\mu = 0.5$ is strongly reduced when $\mu = 0.05$ (not represented here): agents then have more time to make up their mind since opinions are changing 10 times more slowly.

A large number of simulations were carried out and we found that the qualitative dynamics mostly depend on the threshold d. μ and N only influence convergence time and the width of the distribution of final opinions (when a large number of different random sampling are made). d controls the number of peaks of the final distribution of opinions as shown in figure 4. The maximum number of peaks, p_{max}, decreases as a function of d. A rough evaluation of p_{max} based on a minimal distance of $2d$ between peaks (all other intermediate opinions being attracted by one of the peaks), plus a minimal distance of d of extreme peaks from 0 and 1 egdes gives $p_{max} = \frac{1}{2d}$, in accordance with the observations of figure 4.

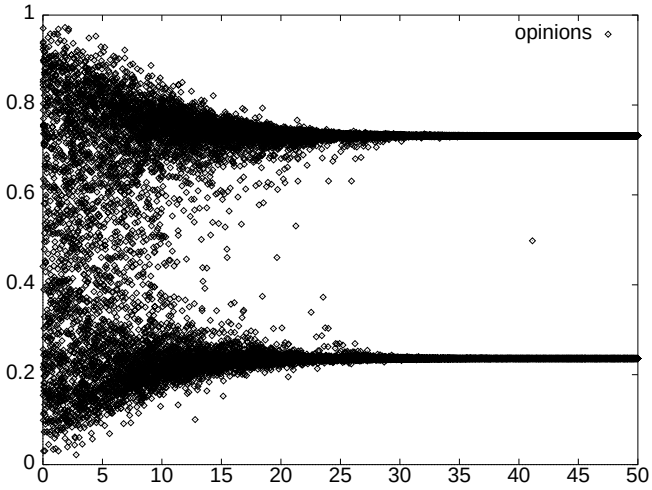

Figure 2. *Time chart of opinions ($d = 0.2 \quad \mu = 0.5 \quad N = 1000$). One time unit corresponds to sampling 1000 pairs of agents.*

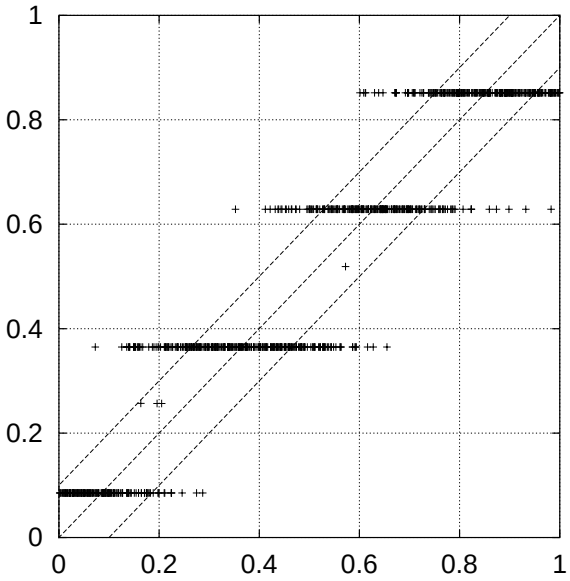

Figure 3. *Diagram of final opinions vs initial opinions ($\mu = 0.5 \quad d = 0.1$)*

The finiteness of the population allows some slight variations of the number of peaks according to random samplings for intermediate values of d. These size effects were confirmed when studying larger and smaller population sizes. In the intermediate regions one also observes small populations wings (a few percent) in the vicinity of extreme opinions 0 and 1.

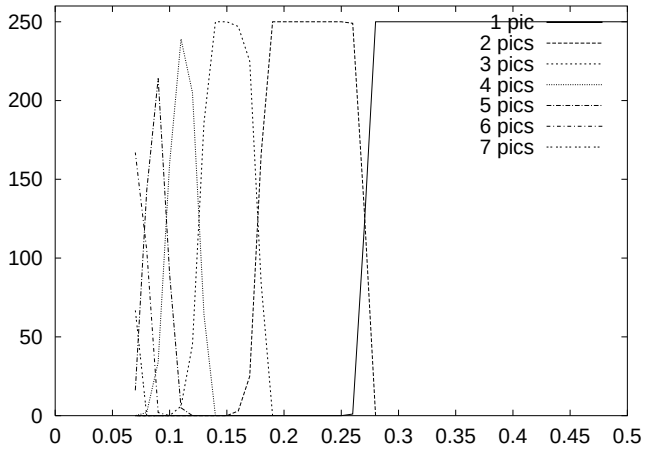

Figure 4. *Statistics of the number of peaks of opinions as a function of d on the x axis for 250 samples ($\mu = 0.5 \quad N = 1000$, wings are excluded from the statistics).*

3. Social Networks

The literature on social influence and social choice also considers the cases when interactions occur along social connections between agents [FOL 74] rather than randomly across the whole populations. Apart from the similarity condition, we now add to our model a condition on neighborhood: agents only interact if they are directly connected through a social pre-existing relation. Although one might certainly consider the possibility that opinions on certain unimportant subjects could be influenced by complete strangers, we expect important decisions to be influenced by advice taken either from professionals (doctors for instance) or from socially connected persons (e.g. through family, business or clubs). Facing the difficulty of inventing a credible instance of a social network as in the literature on social binary choice [WEI 99], we adopted the standard simplification and we carried out our simulations on square lattices: square lattices are simple, allow easy visualisation of opinion configurations and contain many short loops, a property that they share with "real" social networks.

We then started from a 2 dimensional network of connected agents on a square grid. Any agent can only interact with his four connected neighbours (N, S, E and W). We then use the same initial random sampling of opinions from 0 to 1 and the same

basic interaction process between agents than in the previous sections. At each time step a pair is randomly selected among *connected agents* and opinions are updated according to equations 1 and 2 provided of course that their distance is less than d.

The results are not very different from those observed with non-local opinion mixing described in the previous section, at least for the larger values of d ($d > 0.3$). As seen on figure 5 the lattice is filled with a large majority of agents which have reached consensus around $x = 0.5$ apart from isolated agents which have "extremists" opinions closer to 0 or 1: the importance of extremists is the most noticeable difference with the full mxing case described in the previous section.

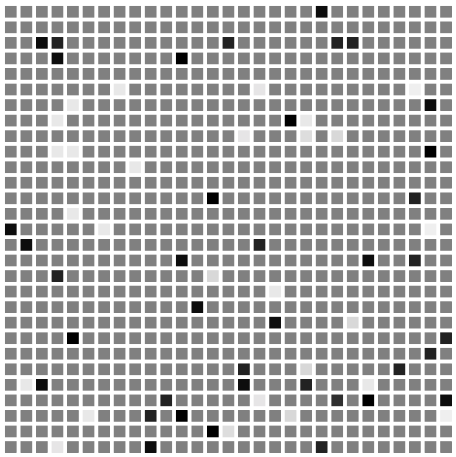

Figure 5. *Display of final opinions of agents connected on a square lattice of size 29x29 ($d = 0.3 \quad \mu = 0.3$ after 100 000 iterations) . Note the percolation of the large cluster of homogeneous opinion and the presence of isolated "extremists".*

Interesting differences are noticeable for the smaller values of $d < 0.3$ as observed in figure 6. When several values are possible for clusters of converging opinions, consensus is can only be reached on connected clusters of agents.

For connectivity 4 on a square lattice, only one cluster can percolate [STA 94] across the lattice which then has homogeneous opinion for all the agents that belong to it. Otherwise, non percolating clusters have homogeneous opinions inside the cluster and these opinions correspond to groups of non-connected clusters with similar but not exactly equal opinions as observed on the histogram (figure 7) and on the pattern of opinions on the lattice (figure 6). The differences of opinions between group of clusters relates to d, but the actual values inside a small cluster fluctuates because homogenisation occurred independently among the different clusters: the resulting opinion depends on fluctuations of initial opinions and histories from one cluster to the other. The same increase of fluctuations versus the full mixing case is observed from sample to sample with the same parameter values.

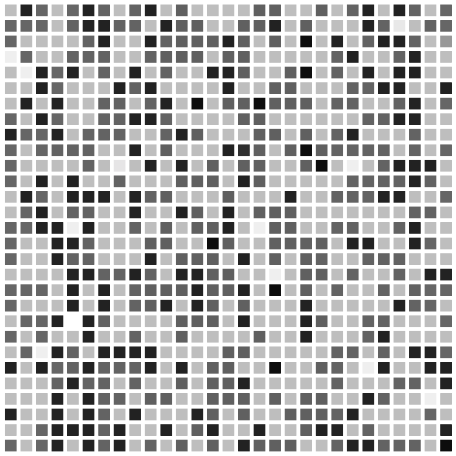

Figure 6. *Display of final opinions of agents connected on a square lattice of size 29x29 ($d = 0.15 \quad \mu = 0.3$ after 100 000 iterations) . One still observes a large percolating cluster of homogeneous opinion and the presence of smaller non-percolating clusters with similar but not equal opinions.*

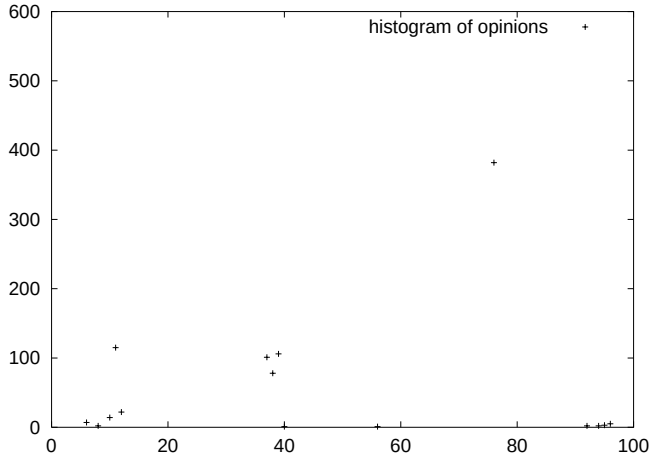

Figure 7. *Histogram of final opinions corresponding to the pattern observed on figure 6 . The 101 bins noted from 0 to 100 correspond to a hundred times the final opinions which vary between 0 and 1.0. Note the high narrow peak at 76 corresponding to the percolating cluster and the two groups of smaller peaks corresponding to non-percolating clusters.*

4. Vector opinions

4.1. *The model*

Another direction for investigation are vectors of opinions. Usually people have opinions on different subjects, which can be represented by vectors of opinions. In accordance with our previous hypotheses, we suppose that one agent interacts concerning different subjects with another agent according to some distance with the other agent's vector of opinions. In order to simplify the model, we reverse to binary opinions. An agent is characterised by a vector of m binary opinions about the complete set of m subjects, shared by all agents. Here, we only treat the case of complete mixing; any couple of agents might interact and adjust opinions according to how many opinions they share. The adjustment process occurs when agents agree on at least $m - d$ subjects (i. e. they disagree on $d - 1$ or less subjects). The rules for adjustment are the following: all equal opinions are conserved; when opinions on a subject differ, one agent (randomly selected in the pair) is convinced by the other agent with probability μ. Obviously this model has connections with population genetics in the presence of sexual recombination when reproduction only occurs if genome distance is smaller than a given threshold; such a dynamics results in the emergence of species (see [HIG 91]).

We are again interested to figure out how opinion vectors cluster. In fact clusters of opinions here play the same role as biological species in evolution. A first guess is that vector opinions dynamics might be intermediate between the binary opinion case and continuous opinions.

4.2. *Results*

We observed once again that μ and N only modify convergence times towards equilibrium; the most influent factors are threshold d and m the number of subjects under discussion. Most simulations were done for $m = 13$. For $N = 1000$, convergence times are of the order of 10 millions pair iterations. For $m = 13$:

– When $d > 7$, the radius of the hyperspace, convergence towards a single opinion occurs.

– Between $d = 7$ and $d = 4$ a similar convergence is observed for more than 99.5 perc. of the agents with the exception of a few clustered or isolated opinions distant from the main peak by roughly 7.

– For $d = 3$, one observes from 2 to seven significant peaks (with a population larger than 1 perc.) plus some isolated opinions.

– For $d = 2$ a large number (around 500) of small clusters is observed (The number of opinions is still smaller than the maximum number of opinions distant by 2).

The same kind of results is obtained with larger values of m: two regimes, uniformity of opinions for larger d values and extreme diversity for smaller d values are

separated by one d value for which a small number of clusters is observed (for $m = 21$ e.g, $d = 5$).

Since there is no a priori reference opinion as in the previous cases of continuous opinions, information about the repartition of opinions is obtained from the histogram of distances among couples of opinions represented on figure 8. The results of the two next figures were obtained by averaging over 200 samples.

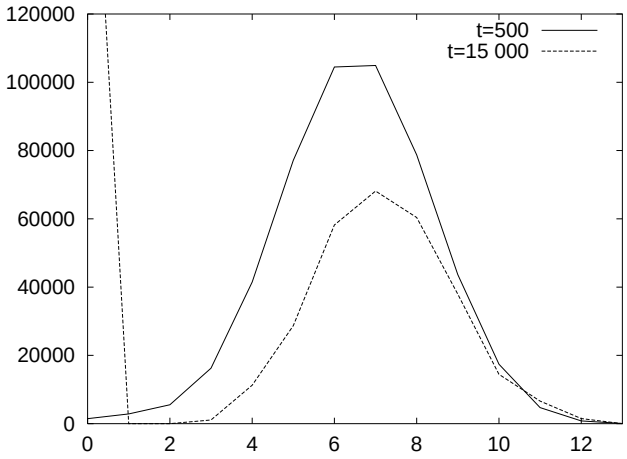

Figure 8. *Histogram of distances in vector opinions for $N = 1000$ agents and thus 500 000 pairs of agents. ($d = 3 \quad \mu = 1$), times are given in 1000 iterations units.*

One important result is that polarisation of opinions (opposite vectors of opinions) is not observed. The clustering process rather results in orthogonalisation of opinions with an average distance around 6: opinion vectors have no correlation, positive or negative, whatsoever. Similar results were observed concerning distances of binary strategies in the minority game [MAR 97].

We were also interested in the populations of the different clusters. Figure 9 represent these populations at equilibrium (iteration time was 12 000 000) in a log-log plot according to their rank-order. No scaling law is obvious from these plots, but we observe the strong qualitative difference in decay rates for various thresholds d.

5. Conclusions

We thus observe than when opinion exchange is limited by similarity of opinions among agents, the dynamics yield isolated clusters among initially randomly distributed opinions. Exchange finally only occurs inside clusters as a the result of the exchange dynamics; initially all agents were communicating either directly or indirectly through several directly connected agents. The concertation process as described here

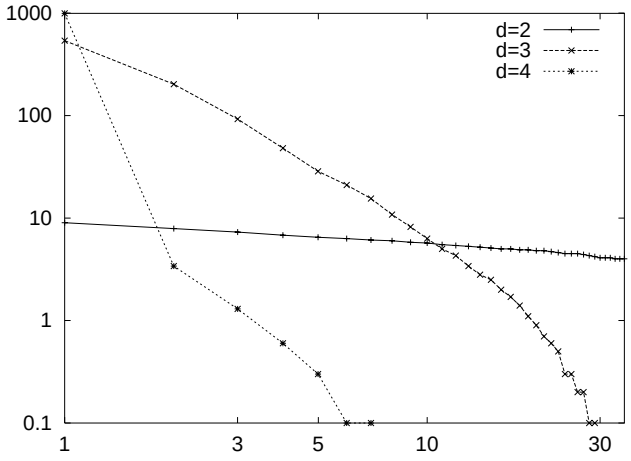

Figure 9. *Log-log plot of average populations of clusters of opinions arranged according to decreasing order. for $N = 1000$ agents ($\mu = 1$).*

is sufficient to ensure clustering even in the absence of difference in private interests or in experience about the external world.

We have studied three very basic models and observed the same clustering behaviour, at least for some parameter regimes, which make us believe that the observed clustering is robust and should be observed in more complicated models, not to mention political life! Many variations and extensions can be proposed, including of course further opinion selection according to experience with some external world ("social reinforcement learning"). An interesting extension would be a kind of historical perspective where subjects (or problems) would appear one after the other: position and discussions concerning an entirely new problem would then be conditioned by the clustering resulting from previous problems.

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