Proceedings of the 2002 SCS *International Conference On Artificial Intelligence, Simulation and Planning in High Autonomy Systems*, Lisbon, Portugal, April 2002, p.253-258

WHICH TIES TO CHOOSE? A SURVEY OF SOCIAL NETWORKS MODELS FOR AGENT-BASED SOCIAL SIMULATIONS

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ABSTRACT

We focus on the social network component of social simulation models. As confronted to the choice of an interaction structure model, "Which ties to choose?" is a relevant question for the social modeller. We present different models of social networks originated from different fields. After presenting a brief ontology about social networks, we classify models into three main parts, the theoretical models coming from physics and mathematics, the statistical models issued from sociology and agent-based models issued from economics, cognitive and computer sciences. The latter focus on a bottom-up approach for modelling social networks.

1 INTRODUCTION

In agent-based simulations, the results depend on which interactions occur between which agents and on which order. Therefore both the choice of agent scheduling and chosen interaction structure (if any) matter (Axtell 2000). In some cases this structure is known a priori, for instance some collective decision-making models where the social network is determined from interviews (Stokman and Van Oosten 1994). Moreover, generally, this structure of the relationships is unknown. One must either determine it or test several hypotheses about its properties. In this paper, we focus on the bestiary of choices about models of interaction structure. As the mean the chosen model influences the simulation outputs depends a lot on the interaction model between the agents, we won't try to quantify or to qualify this effect. We simply propose a state of the art about models of networks developed in several fields, in order to help modellers to choose relevant models concerning their problematic, to test the so-called "social network" effect.

As during the past, social modellers have got the habit to use the cellular automata formalism, the associated interaction structure was most often a regular grid (Schelling 1971) and sometimes classical random graphs (Föllmer 1974). But it yields always to use static structures where social ties do not evolve during simulation. Nowadays, more and more models of social structure, either static or dynamic, have been developed in several fields. These models having different disciplinary origins, we will classify them in function of their original field. Then after introducing an ontology in a first paragraph, we present mathematical and physical theoretical models of networks in a second part. In a third one, we present the work coming from the sociological field. These models are in general statistical models and result from psycho-sociological theories. Finally, we focus on the bottom-up approach, originating from economics, cognitive sciences and computer sciences, aiming at finding individual rules of social networks generation and evolution.

2 ONTOLOGY OF SOCIAL NETWORKS MODELS

In order to clarify the presentation of the different models, we propose a simplified ontology of the domain. To start on, the ontology of social networks models can be seen as a derivation from graph theory (cf. Fig.1) because the structure is similar.

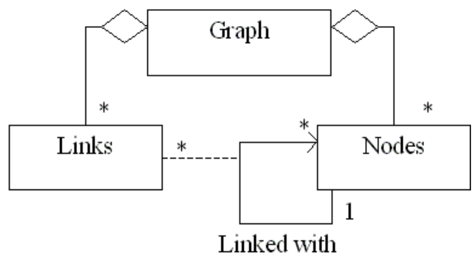

Figure 1: Conceptual diagram of a graph

As a graph is defined by a set of nodes and a set of links between these nodes, a social network is defined as a set of relationships, which are themselves defined as couples of agents or actors (cf. Fig.2).

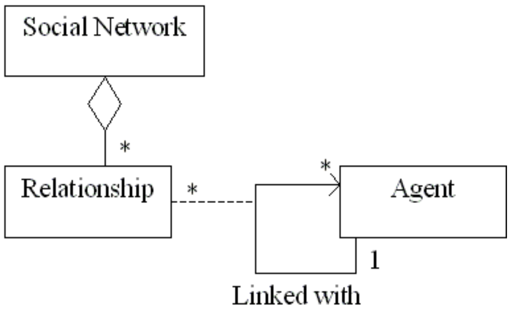

Figure 2: Conceptual diagram of a social network model

The different classes of models differ in the way they generate and apprehend the social network. In theoretical models, the network is seen as an entity by itself generated exogenously without taking into account differences in links types or in agents properties. When constructed iteratively, the only matter is what are the properties of the graph at the precedent time step and which transformation to apply to go to the next one. Agents are then only nodes and relationships among them only simple directed or undirected links in the sense of the graph theory.

In a sociological approach, the network is seen as an aggregation of relationships, often generated using statistical models. The matter is then the generation of the relationships taking into account the agents properties. In an innovation diffusion frame, it may concern the connection between farmer social status (in number of links) and the tendency to adopt the innovation (Valente 1995). Finally, in agent-based approach the network can be seen as a set of relationships generated by the agents themselves. It is necessary then to define the mechanisms of relationship creation, evolution and suppression by the agents themselves.

This classification is quite drastic and it must be underlined that many hybrid approaches exist which mix different generating techniques. For example, many agent-based models initialise the social network using theoretical models. It is necessary also to underline that some works exist on the translation of agent-based models of social network generation to theoretical models using stochastic algorithms (Jin et al. 2001). But the goal of each approach is underlined in this classification, as theoretical models focus on network on a global scale, sociological approach focus on relationships their connection with agent properties and agent-based approach aims at modelling agent's mechanisms of relationships creation and management.

3 THEORETICAL MODELS OF NETWORKS

A huge part of social models came from the physical field with the adaptation of physical laws to the modelling of social systems.

3.1 Cellular automata

The field of social modelling inherited then of many tools coming from mathematics and physics and in particular of cellular automata (Wolfram 1986). The corresponding underlying interaction structure is then in general the one of a regular grid and in many cases a torus. The agents are then represented by the cells of the automata and their social neighbourhood is then defined from the regular grid with a Newman's or a DeMoore's neighbourhood. The Newman's neighbourhood links a cell to its four neighbours (North, East, South, West) as De Moore's neighbourhood adds four more neighbours (NE, SE, SW, NW) (cf. Fig.3).

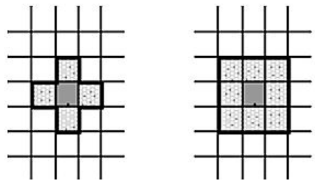

Figure 3: Von Newman's (left) and 3x3 De Moore's neighbourhood (right) on a cellular automata

As a generalization of these kinds of networks, (Flache and Hegselmann 2001) propose interesting tests of their models on different grid related structures like regular, hexagonal and irregular grids determined from Voronoi's diagram (cf. Fig. 4).

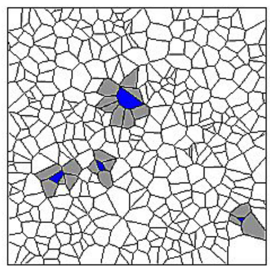

Figure 4: Neighbourhood determined from a Voronoi's diagram.

The main problem encountered within this approach is that you have to start from a tessellation of a two dimensional space. And, whatever this tessellation is, to derive from it, the cells localization and their neighbourhood. And for some social systems that are, for example, not only linked to a geographical space, it can be quite a limitation.

3.2 Random graphs

Some scientists try thereafter to define the interaction structure of their models by using a random graph following the model of (Erdos and Renyi 1960). A random graph is a set of N nodes connected by n edges which are chosen randomly from the $N(N-1)/2$ possible edges (cf. Fig.6 for an illustration). A great advantage of this model compare to the one of Newman or De Moore is that you can change the average connectivity of the graph, as it is fixed to 4 and 8 for the Von Newman's or the 3x3 De Moore's neighbourhood. Concerning grids generated from Voronoi's diagrams, it is also possible to tune the average connectivity within some boundaries, even if it is less easily, but random graph has got this second advantage to suppress the dependence to a two-dimensional space tessellation.

From this stage, many scientists show that there is a "social network" effect depending on the chosen interaction structure characteristics. For example, in the case of communication models, (Carley 1990) exhibits some structural constraints on communication. The fact is that these two classes of networks have got very different characteristics. In term of clustering the regular graphs exhibit more clustering or local redundancy than the random graphs. As a counterpart, random graphs lead to a shortest averaging path length among the couples of individuals than a regular graph. Each one of these characteristics may influence greatly the simulation outputs of an agent-based model (Axtell 2000).

3.3 Small Worlds networks

The arising question at this stage is: do some graphs between these two extremes that may have other characteristics exist? A first class of models comes from the Watts and Strogatz's work about the class of Small World graphs (Watts and Strogatz 1998; Watts 1999). They propose algorithms that enable to fill the gap between regular structures and random graphs. The first model, the α-model, starts from the Erdös and Rényi model but it increases the probability p of link creation in an α proportion of the mutual friends the two agents taken into account have in common. It yields rapidly to cluster formation on the structure. The second one, the β-model is based on a substrate, a starting structure that is regular. Hereafter, depending on the β variable, comprised between 0 and 1, it keeps or rewires at random each link of the existing graph (cf. Fig. 5).

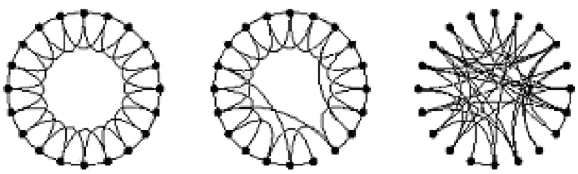

Figure 5: The β-model of (Watts, 1999) enables to go from regular graphs (on the left) to random graphs (on the right) using rewiring of edges.

It yields to the introduction on a regular structure of random graphs characteristics especially the reduction of the average path length among the agents. The third one, the φ-model, is similar to the β-model but adds the criterion that new created links must be shortcuts. The local redundancy of links is then forbidden and the average distance among agents is reduced more rapidly than in the β-model. These Small World graphs inherit in a way of each one of the properties of both regular and random graphs. They have both a high clustering coefficient and a low averaging distance. It follows, for the scientists who test their models on this class of graphs that their models were functioning very differently (Axtell 2000; Soorapanth et al. 2001).

3.4 Scale-free networks

Confronting models to empirical data on several kinds of networks, as the Internet or the metabolic network, Barabasi's team exhibits a power-law distribution of nodes' degree (Albert et al. 1999). It is a property they do not encounter in the existing theoretical models. Then they propose the model of scale-free networks (Barabasi et al. 2000) which has the degree distribution as an input, but is random in all other aspects (cf. Fig. 6). The scale-free model is an iterative one. It starts from a seed of nodes and adding new ones, it adds also corresponding links following the rule that the more a node has got entering links the more it has chance to receive new ones. This probability is tuned by a parameter of the model. Then the model of scale-free networks can be applied in particular to generate random graphs with a degree distribution that follows a power-law.

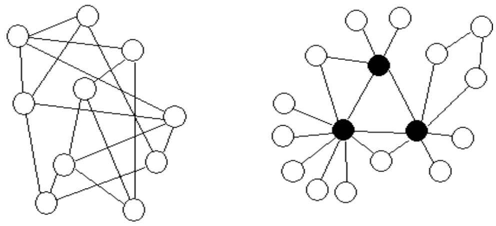

Figure 6: Random and scale-free networks

Within this class of graphs they propose several directions to modify it. In (Albert and Barabasi 2000) they propose a rewiring edges dynamics that enables also links deletion in time. (Dorogovtsev and Mendes 2000) propose in the same way a stochastic algorithm that add edges between old sites and that can remove existing edges, the probability of link creation between two nodes being proportional to the product of the their degrees. At last, (Amaral et al. 2000) proposes an aging cost or a capacity constraint to new link creation that refrain the process of power-law degree distribution.

4 STATISTICAL MODELS OF SOCIAL NETWORKS

In comparison with the models described above, many social scientists chose to build the underlying interaction structure of their models using statistical models. We will focus here on four important models in the domain, as each one of them gave birth to many descendants. They have in common that, given a couple of individuals and their characteristics, they focus on the probability of existence of the relationship among them. It has to be noticed that the major part of models deals with oriented relationships that is not the tendency observed in theoretical models. A good review about these kinds of models can be found in (Banks and Carley 1996).

4.1 The Holland-Leinhardt's models

The Holland-Leinhardt's *p1* model (Holland and Leinhardt 1981) is a log-linear model. This model describes a network in which each dyad (couple of agents) forms edges according to its own probability distribution, depending on the attributes of the couple of agents. The behaviour of one dyad confers no information about the behaviour of any other one. Four parameters specify it, respectively, the agents' tendency to choose other agents, to be chosen by others, to make mutual choices and the average tendency to interact with others.

The *p2* model (Van Duijn and Snijders 1995) is a logistic regression model for social relationships treated as dependant variables with a stochastic component. It derivates from *p1* by adding on the one hand a reciprocity effect and on the other hand a random effect in emitting and receiving relational choices.

Linking *p1* to Markov stochastic graphs, and its estimation to logistic regression methods, (Wasserman and Pattison 1996) propose the *p** model which incorporates structural characteristics of the social network.

4.2 Metric models

The main used method is there to rebuild a corresponding social space, where every agent is localized. Then a stochastic algorithm is used to create the social links depending on a proximity function, in general the Euclidean distance taken on weighted dimensions, between every two selected individuals. Many existing works retain this solution in some way, either by creating social networks that depend only on a geographical distance (Nowak and Vallacher 1998) or by including other attributes, like socio-economical indices of the individuals. A generalisation of the construction of a social metric has been proposed in (Banks and Carley 1994). In has to be noted that the prevalent concept within much of these models is the one of homophily. The more agents are similar, or the closer they are within the social space, greater is their tendency to be linked. A spatial version of the construction of a social space is proposed by (Epstein and Axtell 1996), agents are moving on a map and the individuals they meet incidentally are integrated to their social network. The agents' behaviour is influenced by their social network, and relationships are broken when individuals become to far away one from the other.

4.3 Triad completion models

These models are derived from Heider's work on balance (Heider 1958), where individuals seek to minimize perceived imbalance. Thus, if John considers himself a friend of Jennifer, and John perceives that Jennifer is friend with Bob, then if John is not friend with Bob there will be an imbalance he will try to correct. The focus object is then no more the only dyad but the triad, formed by a three-person group and the relations among them. The aim of the models is to assign a probability for the creation of a new link within the triad, given the existing links. This has led to a series of interrelated models; these include Heider's "balance" model (Heider 1946), "clustering" model (Davis 1967), "transitivity" model (Holland and Leinhardt 1971) and Newcomb "positive balance" model (Newcomb 1968). (Johnson 1986) gives a discussion about these models.

4.4 Degree variance models

Degree variance models derive in part from the work by (Blau 1967) on exchange theory. These models assume that nodes differ in their intrinsic probability of attracting an edge. Nodes with high degree centrality tend to attract many edges, while those with low degree centrality tend to attract few, as for scale-free networks (Barabasi et al. 2000). Polarization and Balkanisation also follow from theoretical conceptions of power, and may be viewed as variations on the degree variance model. Polarization occurs when the society splits into two groups each centred around a specific node, as happens when there are opposite cliques, each with a powerful leader. Balkanisation occurs when the society splits into a large number of groups, each centred around a specific node.

5 AGENTS RELATIONSHIPS LEADING TO NETWORKS: THE BOTTOM-UP APPROACH

The last category of social networks models deals with a bottom-up approach. Its aim concerns the modelling of the individual socio-cognitive mechanisms used to build, entertain and break relationships. We differentiate two approaches, the first one issued from the game theory that pleads for rational mechanisms of links creation. The second one aims at modelling socio-cognitive properties of relationships creation, strongly linked with socio-psychological theories. And given these mechanisms, it aims at observing which kinds of networks it engenders. As within this case, the mechanisms at play in social networks evolution are strongly interrelated with the aim of the model, it is quite difficult to focus on the single object of relationships creation and evolution without taking into account the aim of the model.

5.1 Game theoretical models

In economics, and in game theory in particular, the framework used is the one of the utility function maximization. Each individual has got the same utility function and they may have different resources to invest or change their behaviour. The models take then into account the relationships creation and evolution (Myerson 1977). The creation or maintain of each relationship has got a cost for the agent. And the sharing of gains depends on the structure of the social network (Slikker and van den Nouweland 2001). Then there is a trade-off between constructing the less relationships possible because it is expensive and to build some of them to have the best access to the resources (Bala and Goyal 1999). The aim is then to study the dilemma between stability and efficiency of the generated networks (Jackson 2001).

Another tendency within game theory is to apply classical games and to test them on different social structures where only linked agents can interact. Within this case, economists often use theoretical models of social networks like random graphs, regular structures or small world graphs (Peyton Young 1998).

5.2 Agent-Based Social Simulation

As a second part, the aim of agent-based modelling dealing with social networks is to express individual socio-cognitive mechanisms of relationships creation, entertainment and deletion. The theoretical basis of these mechanisms comes mainly from socio-psychological theories of group behaviour. Some models differ in the way they initialise the social networks, either choosing an empty network (Zeggelink 1993) or either beginning with an existing one which can be chosen from theoretical models or empirical data (Auer and Norris 2001). As (Mosler and Brucks 2001) justify the use of agent-based simulations to explore socio-psychological theories, some examples enlighten this point concerning theories about relationships creation and management. (Flache and Macy 1997) used an agent stochastic learning approach to discuss Homan's exchange theory, (Zeggelink 1993) try existing theories and suggests some new models about friendship networks, at last (Snijders 1996) proposes stochastic actor-oriented models to test Newcomb's fraternity theory.

Concerning the approaches based on a utility function, (Stokman and Van Oosten 1994; Stokman and Zeggelink 1997) develop agent-based models of political negotiations integrating the modelling of political networks, namely two-stage model and dynamic access model.

Within the field of cognitive sciences, the aim is quite the same. Following the Belief Desire Intention framework, the agent-driven dynamics on relationships may come from the need for the agent to build new links in a way to achieve its goals or to more complex processes like the reduction of cognitive dissonance or to attain a strategic position within a network related to information diffusion. Some methodological issues on this point can be found in (Carley 1989). As expressed above, it is not easy to isolate the part of an agent model dedicated to the network dynamics from the target of the model. Anyway, in many agent-based models, a particular attribute plays a role in social networks dynamics, that is trust in other agents, as evocated in (Falcone and Castelfranchi 2001).

In fact some models are of course a mix of some models exposed above, it is the case of (Auer and Norris 2001) mixing random meetings and strategic action on their network.

A great issue within this field is to characterize the obtained network, as (Nowak and Vallacher 1998) use the Hopfield's model of neural network to characterize the attractor on a simple influence model, it is not always possible and it is sometimes useful to examine the simulation trace of network evolution as proposed by (Bousquet et al. 1998) with a social network observer integrated to their multi-agent simulation platform.

6 CONCLUSION AND PERSPECTIVES

The title question "Which ties to choose?" has obviously not been answered within the paper, as it can't be answer in any. As the model of network structure to choose is strongly linked with the targeted system to model, it is clearly a pitfall to try and answer it. The more that we can do is to present a bestiary of models trying to give a depiction to the modeler of what is done by others dealing with this issue. Now the least that social modelers can do, aware as they are that there exist a social network effect, is to wonder if their models remind the same properties when confronted to other social network structures, are the models robust to changes of interaction structures. The next milestone within this work is to propose an architecture of simulation software that takes into account the social network parameter as every other parameter of the simulation and to integrate it to a simulation software.

ACKNOWLEDGMENTS

Many thanks to a tie of mine, Nils Ferrand for helpful references and for finding the title 'Which ties to choose?".

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