Interacting Agents and Continuous Opinions Dynamics

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Abstract: We present several possible extensions of a model of opinion dynamics in which each agent adjusts a continuous opinion as a result of an interaction with a randomly chosen other agent whenever the difference between their opinions is below a given threshold. In this first model, high thresholds yield convergence of all opinions towards the average opinion, whereas low thresholds result in several opinion clusters. We then focus on a particular case where the threshold can take two values : a low and high one. If the thresholds remain constant, different clustering time scales are observed: on the long run clustering depends on the higher threshold. If we interpret the threshold as an uncertainty of the opinion, we define dynamics of influence on the uncertainty as well as on the opinions when agents interact. In this case, opinion clustering is driven by threshold dynamics. In particular, we study the case where confident (low uncertainty) agents have more influence and are located at the extremes of the distribution (the extremists). We study the parameter values for which the extremists are dominant and lead to the constitution of two opposite clusters by attracting all the moderate agents.

Keywords: agent-based modelling, opinion dynamics, uncertainty, heterogeneous agents

JEL: C6, D8, Q1

Introduction

Many models about opinion dynamics [Föllmer 1974], [Arthur 1994], [Orléan 1995], [Latané and Nowak 1997], [Galam 1997], [Weisbuch and Boudjema 1999], are based on binary opinions which social actors update as a result of social influence.

Binary opinion dynamics under imitation processes have been well studied, and we expect that in most cases the attractor of the dynamics will display uniformity of opinions, either 0 or 1, when interactions occur across the whole population. This is the ``herd'' behaviour often described by economists [Föllmer 1974], [Arthur 1994], [Orléan 1995]. Clusters of opposite opinions appear when the dynamics occurs on a social network with exchanges restricted to connected agents. Clustering is reinforced when agent diversity, such as a disparity in influence, is introduced, [Latané and Nowak 1997], [Galam 1997], [Weisbuch and Boudjema 1999].

One issue of interest concerns the importance of the binary assumption: what would happen if opinion were a continuous variable such as the worthiness of a choice (a utility in economics), or some belief about the adjustment of a control parameter?

The a priori guess for continuous opinions is also homogenisation, but converging towards average initial opinion [Laslier 1989].

The purpose of this paper is to present several extensions of a first model presented in [Deffuant et al. 2000]. In this first model agents with continuous opinions interact only when the difference between their opinions is below a given threshold. The rationale for the threshold condition is that agents only interact when their opinion are already close enough; otherwise they do not even bother to discuss. The reason for such behaviour might be for instance lack of understanding, conflicts of interest, or social pressure. The threshold would then correspond to some openness character. Another interpretation is that the threshold corresponds to uncertainty: the agents have some initial views with some degree of uncertainty and would not care about other views outside their uncertainty range.

We will here give results concerning complete mixing across the whole population (every agent is connected to all the others). We first recall the results obtained in the case of a single threshold. Then, we focus on the case where the threshold can initially take two values: a low and a high one. We first present some results for the case where the threshold remains constant, and then several possibilities for adjusting the thresholds during the interactions. In the last sections, we interpret the thresholds as uncertainties, and we define a dynamics in which more certain agents have more influence. We study in more details specific initial distributions in which confident (low uncertainty) agents are located at the extremes of the distribution.

The basic case: Complete Mixing and one fixed threshold

Let us consider a population of N agents i with continuous opinions x_i. We start from an initial distribution of opinions, most often uniform on [-1,1] in the computer simulations. At each time step any two randomly chosen agents meet. They re-adjust their opinion when their difference of opinion is smaller in magnitude than a threshold d. Let's agents' opinions be x and x', when $|x - x'| < d$ agents' opinions are adjusted according to:

$$x = x + \mu(x' - x)$$

$$x' = x' + \mu(x - x')$$

where μ is the convergence parameter whose values may range from 0 to 0.5.

In the basic model, the threshold d, is taken as constant across population.

The evolution of opinions may be mathematically predicted in the limiting case of small values of d [Neau 2000]. Density variations $\delta\rho(x)$ of opinions x obey the following dynamics:

$$\delta\rho(x) = \frac{-d^3}{2}\mu(1-\mu)\frac{\partial(\rho^2(x))}{\partial x^2}$$

This implies that starting from an initial distribution of opinions in the population, any local higher opinion density is amplified. Peaks of opinions increase and valleys are depleted until very narrow peaks remains among a desert of intermediate opinions.

Computer simulations show that the distribution of opinions evolves towards clusters of homogeneous opinions (at large times). For large threshold values (d > *0.6*) only one cluster is observed at the average initial opinion. Figure 1 represents the time evolution of opinions starting from a uniform distribution of opinions.

3D graphs represent the evolution of histograms of "opinion segments" defined from opinion x and threshold d by [x-d, x+d]. The z axis measures the number of agents which opinion segment include opinion x given along the x axis.

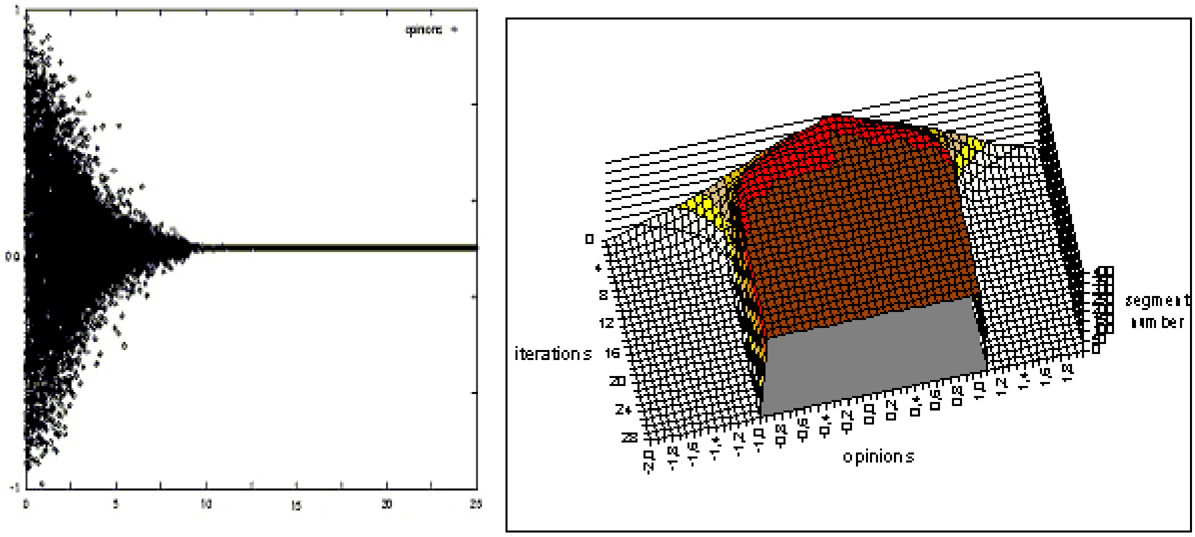

Figure 1: Time chart of opinions (d = 1, μ = 0.5 N = 2000). One iteration corresponds to N interactions between two agents.

For lower threshold values, several clusters can be observed: their number scales as the integer part of $1/d$, to be later referred to as the "1/d" rule (see [Deffuant et al. 2000] for more details).

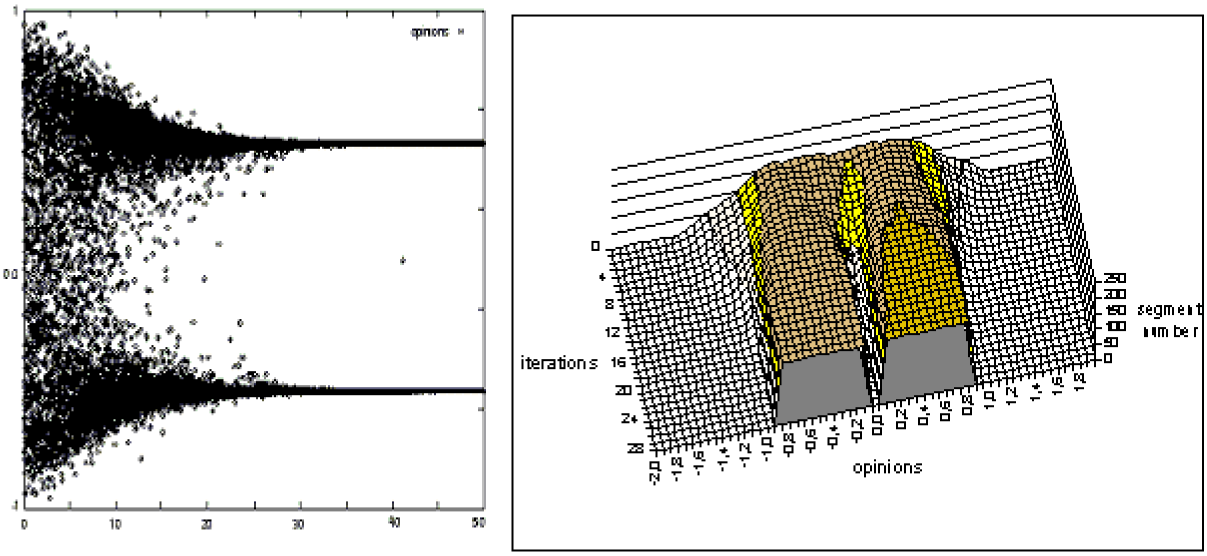

Figure 2: Time chart of opinions for a lower threshold (d = 0.4 μ = 0.5 N = 1000). One time unit corresponds to N interactions between two agents.

In the case of interactions across a social network, the number of clusters is increased; clustering can even get extreme in the case of vectors of opinions (see [Deffuant et al. 2000] for more details).

Two fixed thresholds

Of course, supposing that all agents use the same threshold to decide whether to take into account the views of other agents is only an approximation. When heterogeneity of thresholds is introduced, some new features appear. To simplify the matter, let us take the case of a bimodal distribution of thresholds, for instance 8 agents with a large threshold of 0.8 and 192 with a narrow threshold of 0.4 as in figure 3.

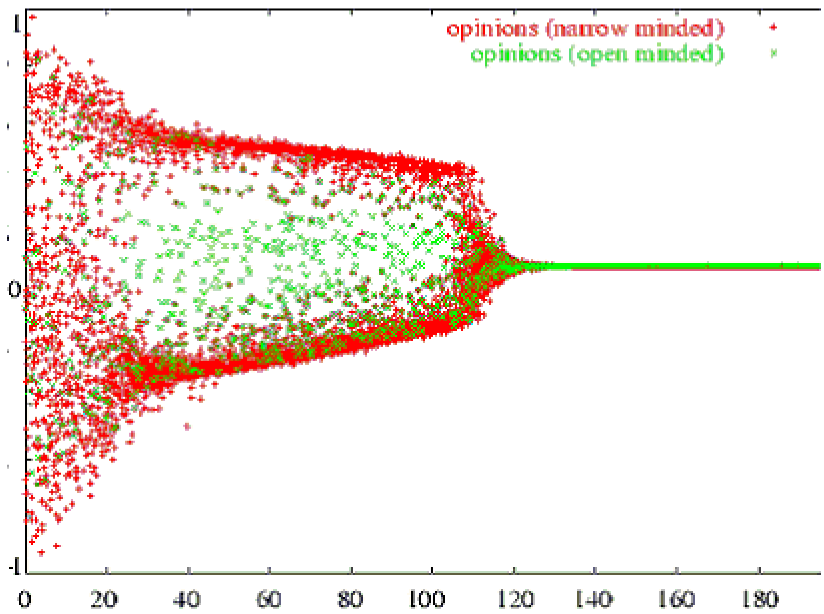

Figure 3 : Time chart of opinions (μ = 0.5 N = 100). Red '+' represent narrow minded opinions (192 agents with threshold 0.2), green 'x' represent open minded opinions (8 agents with threshold 0.4). One time unit corresponds to N interactions between two agents.

One observes that in the long run, for times larger than 12000, convergence of opinions in one single cluster is achieved due to the presence of the few "open minded" agents. But in the short run, a metastable situation with two large opinion clusters close to opinions 0.35 and 0.75 are observed due to the narrow minded agents, with open minded agents opinions fluctuating around 0.5 due to the interactions with narrow minded agents belonging to either high or low cluster. Because of the few exchanges with the high uncertainty agents, confident agents opinions slowly shift towards the average until the difference in opinions between the two clusters becomes lesser than the low threshold: at this point the two clusters collapse. This behaviour is generic for any mixtures of thresholds:

- Clustering in the long run depends on the higher threshold;
- Clustering in the short run depends on the lower threshold;

The transition time depends upon the total number of agents and on the ratio of narrow minded agents to the open minded ones.

Decreasing uncertainty with interactions

If we interpret the threshold as the agent uncertainty, we might suppose with some rationale that subjective uncertainty decreases with the number of opinion exchanges. Any opinion exchange can be interpreted in an information diffusion context, at least by the agent: taking opinions from other agents could be considered as updating information in which case agents would decrease the subjective uncertainty. We then modelled this decrease of threshold d when exchanges occur according to the following equation:

$$d = d - v\frac{1}{|x'-x| + 0.1} \qquad (1)$$

Where v is the threshold decay constant. This expression gives rise to an approximately exponential decay of thresholds.

Initial opinions and exchanges have more importance than later exchanges in determining the outcome of the dynamics (long term memory). We could of course have chosen a more sophisticated algorithm, such as a Bayesian updating [Arthur 1994] or a short term memory [Weisbuch et al. 1996], but we had no sound rationale to choose among these possibilities.

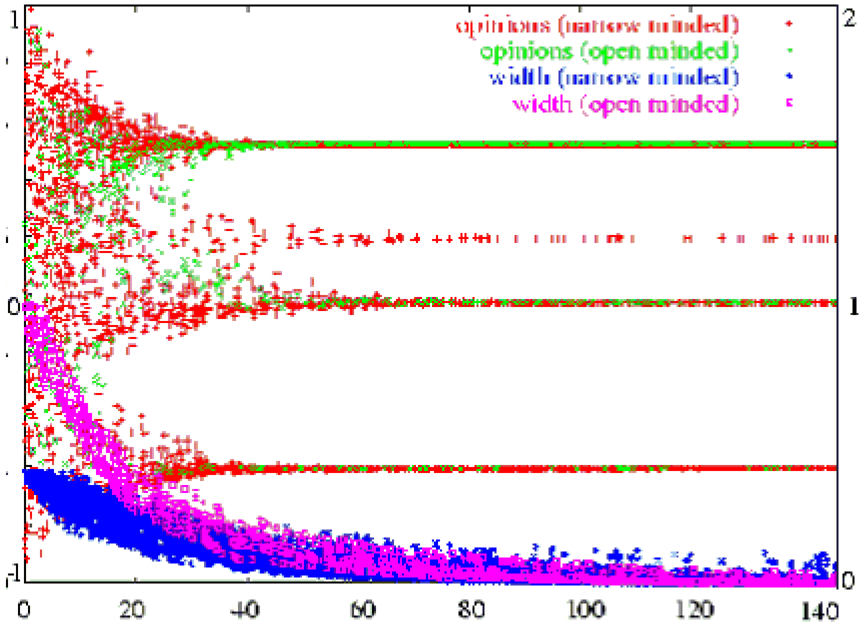

Figure 4 : Time chart of opinions and thresholds (μ = 0.5 ν = 0.05 N = 100, initial large threshold 1 for 10 agents, low threshold 0.4). Violet '+' represent narrow minded thresholds, pink 'x' represent open minded thresholds. Vertical left scale for opinions varies from -1 to +1, right scale for thresholds varies from 0 to 2.

The decrease in uncertainty in connection when exchanges occur results in more clustering than when thresholds are maintained at their initial value. One observes in figure 4 that for 10 agents with an initial threshold of 1.0, and 90 agents with an initial threshold of 0.4, four clusters are maintained at large time.

One can of course wonder whether or not agents would sometimes increase their uncertainty when they consistently encounter agents whose opinions are outside their "tolerance". We haven't investigated this direction, but Epstein's paper [Epstein 2000], in a different context, provides an interesting trail.

Higher and Lower expectations

Another way to define the dynamics is to consider the minimum and maximum values of the segment of opinion. This view can be inspired by the theory of higher and lower expectations which has been developed in statistics and economics. It can be used for real valued uncertain reward x about which people make “gambles”. The lower prevision of a gamble is a value which is interpreted as the highest buying price for the considered quantity. It means that it is acceptable to pay any price smaller than it for the uncertain reward. The higher expectation is interpreted as the lowest selling price for the quantity. This approach has application in any field of risky investments. The difference between the higher and the lower anticipations can be interpreted as the uncertainty about the outcome of the decision. The closer are the lower and higher expectations, the lower the uncertainty is.

One can imagine new dynamics of interactions, in which the agents influence each other’s higher and lower expectations when they interact.

To be more specific, if agent's A opinion x averaged over its high and low values falls within the range of the other agent A' expectations x'_h and x'_l, the modifications δx_l and δx_h of A higher x_h and lower x_l expectations are given by the procedure:

$$\delta x_l = \mu.h.(x_l' - x_l)$$

$$\delta x_h = \mu.h.(x_h' - x_h)$$

Figure 5 exemplifies the exchange of the high and the low opinions for $\mu = 0.5$ when the overlap condition is fulfilled for both set of opinions. Note that in this case the distance between the high and the low opinions simply becomes the average distance of the initial distances of both agents.

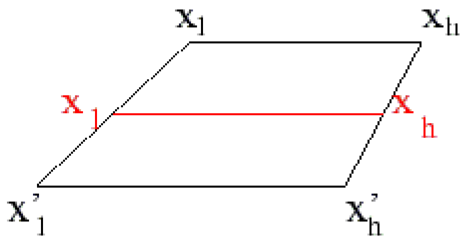

Figure 5: Opinion exchange when $\mu = 0.5$ and when both overlap conditions are fulfilled. Initial opinions are represented in black, final opinions in red.

Figure 6 represents the evolution of opinions for these dynamics. Note the convergence of the distance of high and low opinions toward the average distance, as explained by the trapeze rule represented on figure 5 and the resulting convergence towards three clusters.

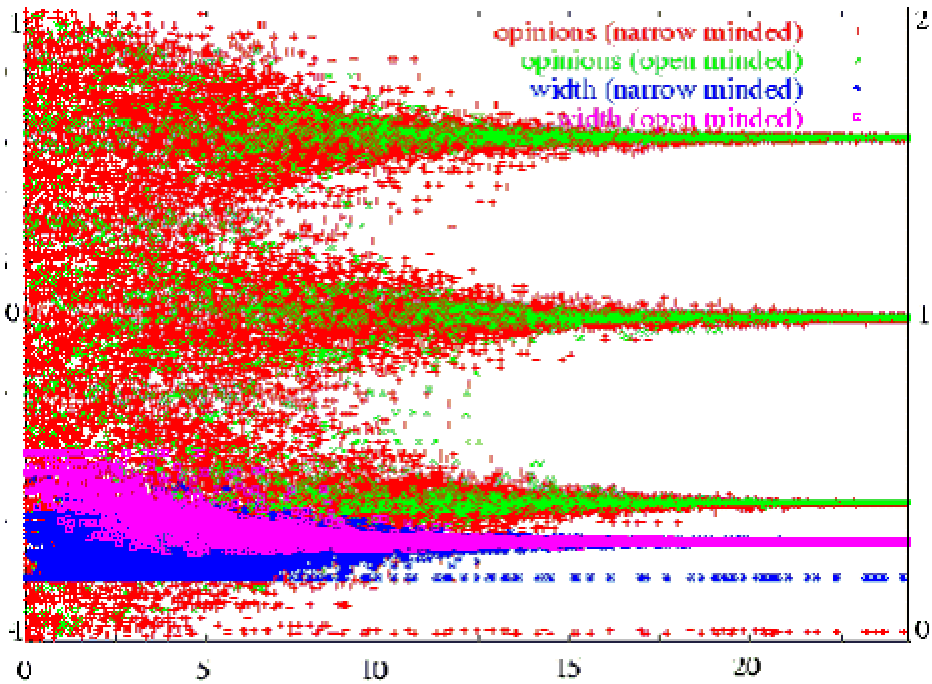

Figure 6 : Time chart of opinions and thresholds ($\mu = 0.5$ $N = 2000$, initial large threshold 0.6 for 100 agents, low threshold 0.2). The dynamics is the high and low opinion dynamics, but the colour conventions are the same as for previous figures. Vertical left scale for opinions varies from -1 to +1, right scale for thresholds varies from 0 to 2.

Giving more influence to "confident" agents

In the previous dynamics, uncertain agents (initialised with the high threshold) have the same influence than confident agents (initialised with the low threshold). This is somehow counter intuitive, because one would expect that confident people tend to have more influence than uncertain ones. According to this interpretation, certainty leads to convincing, and convinced people are more influential than unconvinced people.

Moreover, in the previous dynamics, the middle point of the uncertainty segments plays a particular role which is a bit artificial. For instance, a very large segment has a very high influence on a very small segment located close to the middle of the large segment. If this small segment is slightly shifted (of its length) from the middle of the large segment, it is not influenced anymore. This discontinuity is artificial.

This led to the definition of a new variant of the dynamics which avoids these difficulties.

Let us suppose that agent *A* encounters agent *A'*. Let

- x_l and x_h be the lower and higher expectations of agent *A* before the interaction,
- x'_l and x'_h be the lower and higher expectations of agent *A* before the interaction.

We define mathematical rules for the modification of these expectations after the interaction.

We consider the value *h*, which is the relative overlap between the two intervals of expectations over *A'* expectation interval width :

$$h = \frac{\min(x_h', x_h) - \max(x_l', x_l)}{x_h' - x_l'}$$

If $h > \frac{1}{2}$, we consider that the overlap represents a significant part of *A'* interval width, and the influence takes place. The influence of *A'* on *A* is proportional to *h* and to the difference of the expectations. Let δx_l and δx_h be the adjustments in opinions after the exchange:

$$\delta x_l = \mu.h.(x_l' - x_l)$$

$$\delta x_h = \mu.h.(x_h' - x_h)$$

If $h \leq \frac{1}{2}$ the expectations of *A'* are too incompatible with the ones of *A*, and *A'* has no influence on *A*.

In this model, the confident agents are more influential, because for them, *h* tends to be higher. Moreover, the discontinuity in the interactions that we identified with our previous dynamics is no more present because a very large segment never has an influence on a very small one (as long as the length of the large one is more than twice the length of the small one).

We now explore the properties of this model in the particular case of confident agents located on the extremes of the distribution.

Confident agents located at the extremes

We now suppose that the confident agents are the ones which have the most positive or negative mean opinions. This hypothesis can be justified by the fact that often people who have extreme opinions tend to be more convinced. On the contrary, people who have moderate initial opinions, often express a lack of knowledge and uncertainty.

We define two categories of agents : the extremists, which are initialised with the low uncertainty and are at the extremes of the distribution, and the uncertain agents which have a high uncertainty and are located in the middle of the distribution (see figure 7). The initial distribution is therefore defined by : u the low uncertainty, U the high uncertainty, p_e the proportion of extremists in the population.

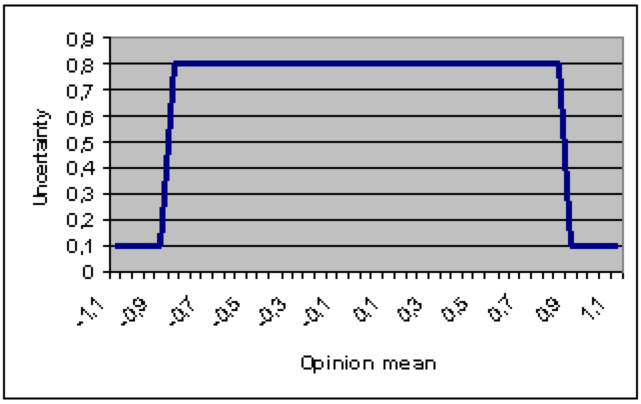

Figure 7 : We suppose that the uncertainty is function of the mean opinion. The mean opinion uniformly distributed between –1 and +1. The proportion of extremists p_e=5% with the low uncertainty u=0.1 and for the uncertain U=0.8.

With these hypotheses, three extreme behaviours of the model can happen :

- the extremists do not influence the normal clustering of the uncertain agents, which for large high uncertainty converge toward 0,
- the extremists modify the clustering and attract the uncertain agents on the extremes, leading to two opposite groups (one positive and one negative) with low uncertainty,
- the extremists attract the whole population on one of the extremes (positive or negative).

Figures 8, 9 and 10 illustrate these behaviours. In these figures, the agents opinion and uncertainty is represented by its opinion segment $[x_l, x_h]$. In the graphs, we draw the histograms of agent's opinion segment as a function of time. The vertical axis represents the number of segments which are present on the considered location.

In order to get a better evaluation of the population evolution, we calculate the average distance d of the segments to 0. This is the distance from 0 to the nearest edge of the segment, counted positively when 0 falls inside the segment, and negatively otherwise. The distance is then divided by the half length of the segment. The difference between the initial and final (after convergence) average distance to 0 expresses the influence of the extremists : when this value is negative, the global population is further from 0 than initially, when it is positive, it expresses that the population converged toward 0. More precisely, the average distance of a segment to 0 is given by :

If $x_l.x_h < 0$, then : $d([x_l, x_h], 0) = \dfrac{2.\min(-x_l, x_h)}{x_h - x_l}$

Else : $d([x_l, x_h], 0) = \dfrac{-2.\min(|x_l|, |x_h|)}{x_h - x_l}$

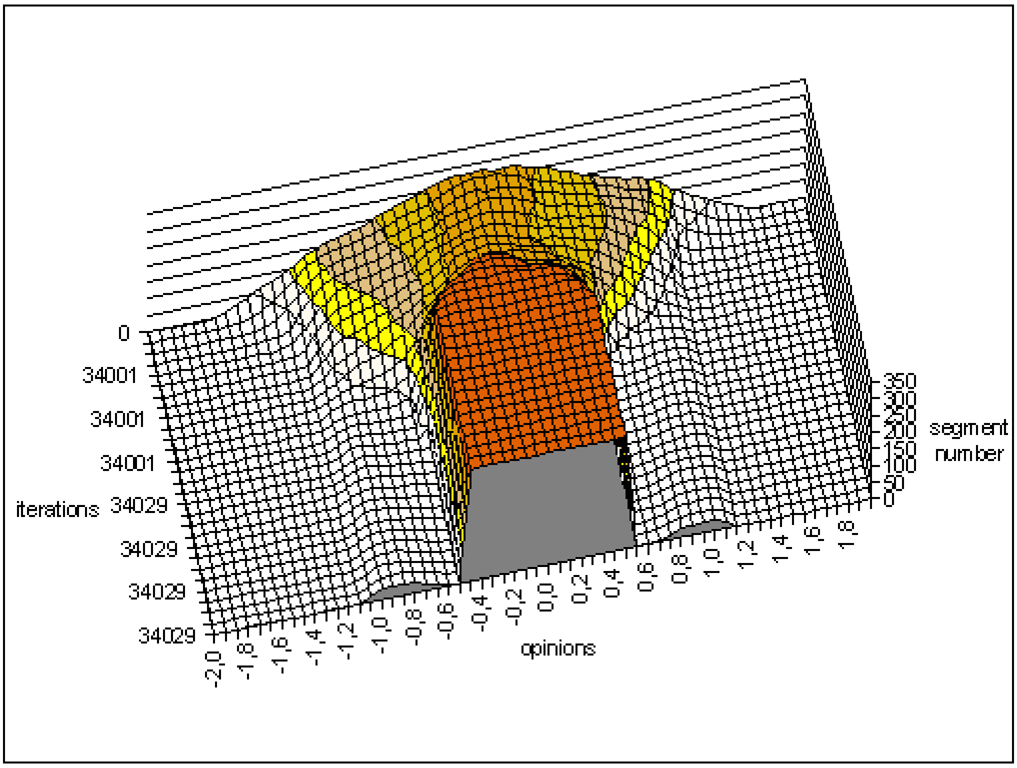

Figure 8 : Example of convergence of the majority toward 0 (with 2 small peaks of extremists). The initial distribution of mean opinions is uniform betwen –1 and 1. N= 400 , small uncertainty u= 0.1, large uncertainty U = 0.4, proportion of extremists p_e *=20%,* μ *= 0.5. The difference between the initial and final average distance to 0 is +0.36. For these parameters the behaviour is not stable : sometimes the second type of behaviour happens.*

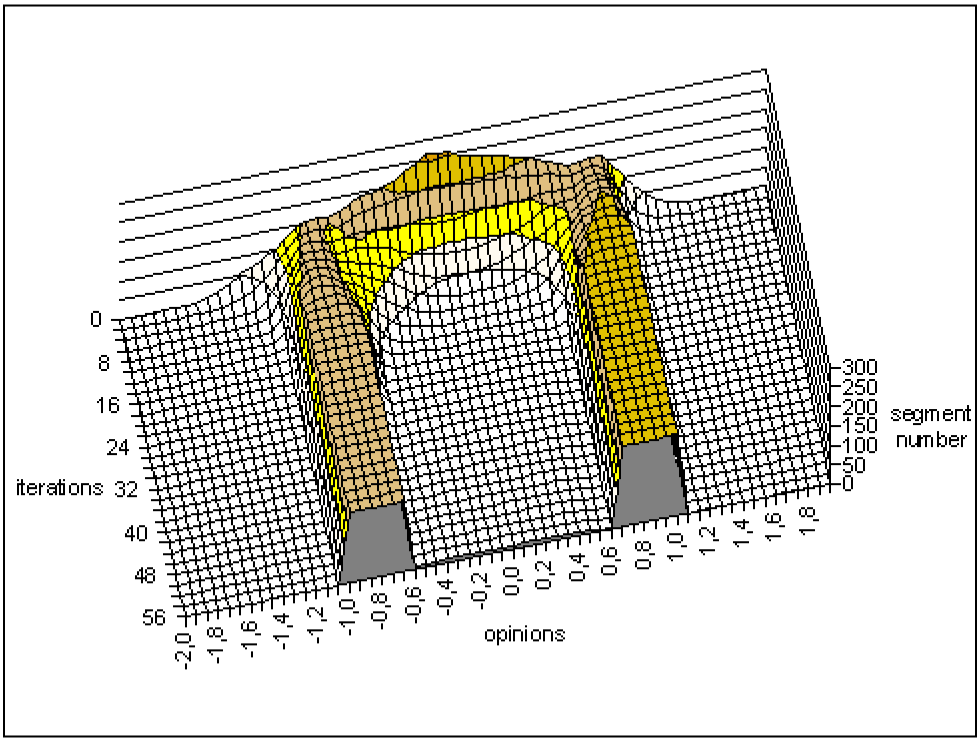

Figure 9 : Example of convergence toward toward both extremes. The distribution of mean opinions is uniform betwen –1 and 1, . N= 400 , small uncertainty u= 0.1, large uncertainty U = 0.7, proportion of extremists p_e *=20%,* μ *= 0.5. The difference between initial and final average distance to 0 of the segments is –2.08. The value of the difference is very stable for these parameters (no variations for different trials of the initial population).*

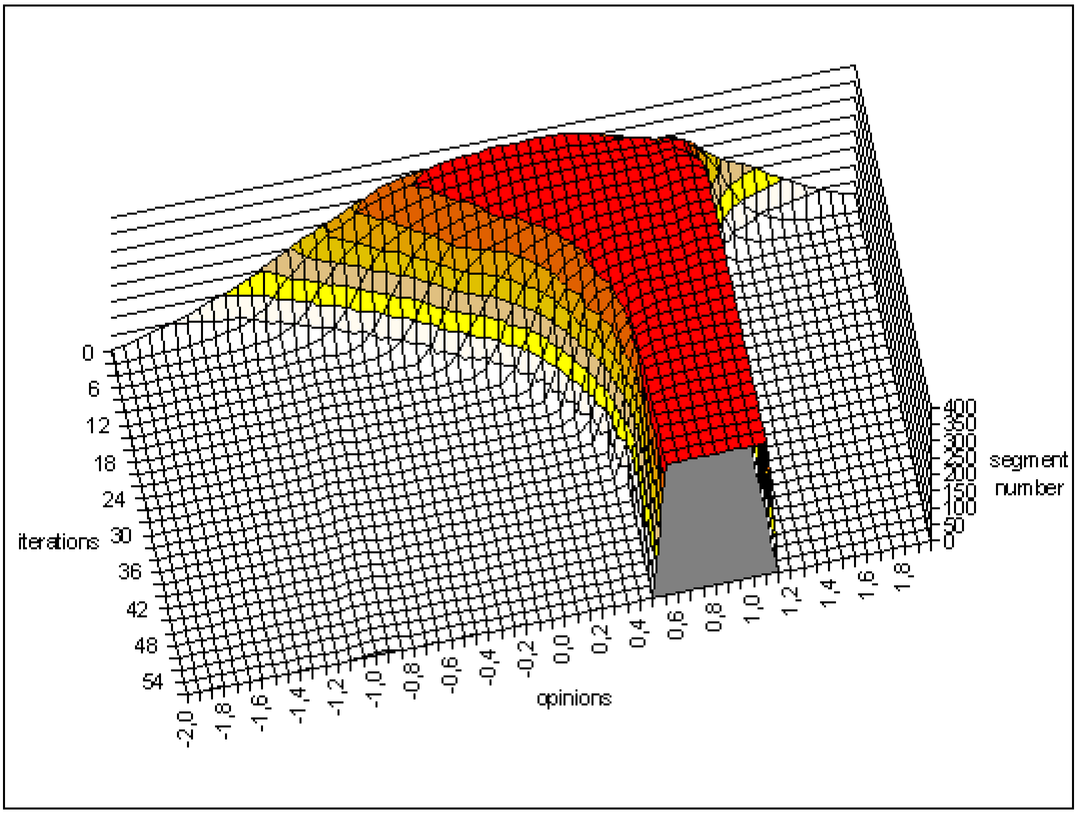

Figure 10 : Example of convergence toward one of the extremes. The distribution of mean opinions is uniform betwen –1 and 1 . N= 400 , small uncertainty u= 0.1, large uncertainty U = 1.2, proportion of extremists p_e =5%, μ = 0.5. The difference between initial and final average distance to 0 of the segments is –1.94. The value of the difference is not very stable for these parameters (for some trials a convergence to the middle happens).

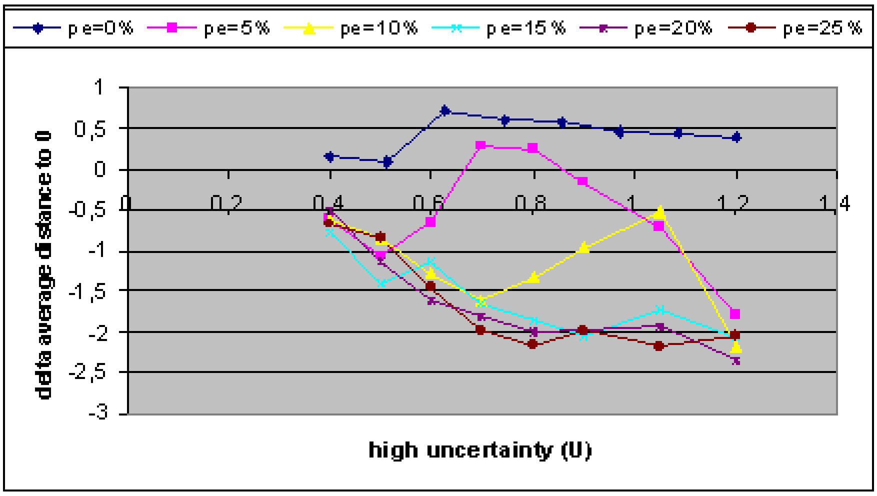

Figure 11 : Plot of the difference between initial and final average distance to 0 for different values of the fraction of extremists p_e (with N=400, u=0.1) versus the large uncertainty U which varies from 0.4 to 1.2, μ = 0.5. Each dot corresponds to averaging over 5 simulations. The initial distribution of mean opinions is uniform. between -1 and 1. We note that when the proportion of extremists is higher than 10%, curves are very similar : the extremists clearly win as soon as the U is higher than 0.6. Below 10% of extremists the influence of the extremists win for higher values of U (1.2) for which there is a convergence toward only one of the extremes.

Figure 11 shows difference between the initial and final average distance to 0 for different values of the parameters. We note from these curves that there is a critical value for the proportion of extremists p_e (around 10%) at which the global behaviour of the model changes.

- For p_e above the critical value, the impact of the extremists does not depend on p_e: it increases when U increases (similar decreasing negative curves for the delta average distance to 0).
- For p_e below the critical value, the delta average distance to 0 increases and then decreases for very large values of U. This decrease corresponds to the convergence to one of the extremes of the whole population.

Conclusion

The basic lessons from these set of simulations is that most often the dynamical behaviour of mixed or evolving thresholds systems can be inferred from the basic single and fixed threshold system, except when we introduced a higher influence for confident agents.

For the first models, the "1/d" rule applies with some precaution. The long term behaviour depends on the final values of evolving thresholds or on the larger threshold for a distribution of fixed threshold. The short term behaviour, which might last for some significant time in term of a socio-economic interpretation is determined by the threshold of the most numerous population or by the convergence of the thresholds towards their attractor. The exceptions to the generic behaviour might occur when some low populations (i.e. a few individuals) allow for fluctuations which might end up into the annihilation of some expected clusters.

When the confident agents have extreme opinions (the extremists) and are more influential, the situation is different. There is threshold value for the fraction of extremists which rules the behaviour of the model. The basic rule is that the extremists' influence increases as the other agents' uncertainty.

Acknowledgments

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