The drift to a single extreme appears only beyond a critical connectivity of the social networks

Study of the relative agreement opinion dynamics on small world networks

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1. Introduction

In (Deffuant et al., 2002) we proposed an agent-based simulation model of opinion dynamics, the relative agreement model (RA model). Its main characteristics are that agents' opinions are continuous rather than discrete or even binary in classical models; that the opinion dynamics of an agent takes into account opinions from others in a limited zone, the agent's uncertainty, around its own opinion as for instance in (Hegselmann and Krause, 2002; Deffuant et al., 2001; Weisbuch et al., 2002a; Weisbuch et al., 2002b). Moreover, the influence takes into account the overlap between the two opinions segments and each other's uncertainty. A property of this model is that the less uncertain is the agent, the more convincing or influential it is. We then introduce a part of convinced agents in the population at the extremes of the initial opinion distribution, the extremists by definition, and we studied the influence of these local attractors on the global opinion dynamics. Three kinds of convergence appear within the model, the initially moderated agents, located around the center of the initial opinion distribution having an initially higher uncertainty than the extremists and being then more influenced than these latter. These moderate agents evolve then either forming a central cluster, being not influenced by the extremists; or they are split in two, each part following one of the extreme; or only one of the extreme attracts the whole population. We studied this model with the assumption that every pair of agents could interact (Deffuant et al., 2002); the population was therefore totally connected. In this paper we present the study of this model when choosing another kind of interaction structure.

In a first paragraph we present briefly the Relative Agreement Model, the addition of extremists within the population and the main results corresponding to the totally connected case. We then describe the first simulations done on a regular grid with a Von Neumann and a De Moore neighborhood. Significant changes occur in comparison with the totally connected case as for instance the disappearance of the single extreme convergence. Aiming at understanding it, we then explore the behavior of the model between regular grids and fully connected networks. We use a Small-World topology that enables us to tune both the average connectivity of the network and the level of noise within the network, from a regular network to a totally random one. The simulations show that there is a critical level of connectivity beyond which the drift to a single extreme appears. We then discuss the observed behavior of our model.

2. The RA Model

2.1. Model definition

We consider a population of N agents. Each agent i is characterized by two variables, its opinion x_i and its uncertainty u_i, both being real numbers. We call segment $s_i = [x_i - u_i, x_i + u_i]$ of the opinion axis the opinion segment of agent i. In the following, we draw the opinions from a uniform distribution between –1 and +1.

We suppose that random pair interactions take place among the agents, during which they may influence each other's opinion segment.

More precisely, let us consider opinion segments $s_i = [x_i - u_i, x_i + u_i]$ and $s_j = [x_j - u_j, x_j + u_j]$. We define the agreement of agent i with j (it is not symmetric) as the overlap of s_i and s_j, minus the non-overlapping part. The overlap h_{ij} is given by:

(Eq. 1) $$h_{ij} = \min(x_i + u_i, x_j + u_j) - \max(x_i - u_i, x_j - u_j)$$

The non-overlapping width is:

(Eq. 2) $$2.u_i - h_{ij}$$

The agreement is the overlap minus the non-overlap:

(Eq.3) $$h_{ij} - (2.u_i - h_{ij}) = 2\cdot(h_{ij} - u_i)$$

The relative agreement is the agreement divided by the length of segment s_i:

(Eq. 4) $$\frac{2\cdot(h_{ij} - u_i)}{2\cdot u_i} = \frac{h_{ij}}{u_i} - 1$$

If $h_{ij} > u_i$, then the modifications of x_j and u_j by the interaction with i are multiplied by the relative agreement:

(Eq. 5) $$x_j := x_j + \mu\cdot\left(\frac{h_{ij}}{u_i} - 1\right)\cdot(x_i - x_j)$$

(Eq. 6) $$u_j := u_j + \mu\cdot\left(\frac{h_{ij}}{u_i} - 1\right)\cdot(u_i - u_j)$$

Where μ is a constant parameter which amplitude controls the speed of the dynamics.
If $h_{ij} \le u_i$, there is no influence of i on j.
The main features of the relative agreement model are:

- During interactions, agents not only influence each other's opinions but also each other's uncertainties.
- The influence is not symmetric when the agents have different uncertainties; "confident" agents (low uncertainty) are more influential.

The influence (the modifications of x_j and u_j) varies continuously when x_j, u_j, x_i and u_i vary continuously.

2.2. Addition of extremists

We now introduce extremists within our population: we suppose that these agents at the extremes of the opinion distribution are more confident. We define therefore two values, on the one hand, their initial uncertainty: u_e the uncertainty of all the extremists, and on the other hand, U the uncertainty of the moderate, supposed bigger than u_e.
We define also p_e as the global proportion of extremists in the population. p_+ and p_- are then the proportion of extremists at the positive or negative extreme opinions.
The relative difference between the proportion of positive and negative extremists is noted δ: *(Eq. 7)*

$$\delta = \frac{|p_+ - p_-|}{p_+ + p_-}$$

In practice, we first randomly draw opinions of $(1-p_e).N$ agents of the population from a uniform distribution between –1 and +1. Then we initialize Np_+ agents to *+1* and Np_- most negative opinions to *–1,* moreover we initialize them with the uncertainty u_e, and the others with the uncertainty U.

2.3. Results on the totally connected case

2.3.1. The attractors: bipolarization or single polarization

This very simplified model of extremism, exhibits different dynamical regimes. We observed parameter zones in which the extremists have a small influence on the rest of the population, and other zones where on the contrary, the majority of the population becomes extremist, either in both extremes or, in one single extreme.
The following set of figures, obtained from numerical simulations, exhibits the three different dynamical regimes. The x-axis codes for time (number of iterations), the y-axis for opinions, and the colors for uncertainty. Each trajectory allows following the evolution in opinion and uncertainty of one individual agent. Common parameters are, $\mu = 0.5$, $\delta = 0$, $u_e = 0.1$, $N = 200$. The uncertainty parameter U of the general population is increased from figure 1 to 4.
Figure 1, obtained for $U = 0.4$, shows an example of central convergence. In this case, only a marginal part of the initially non-extremists became extremist (4%).

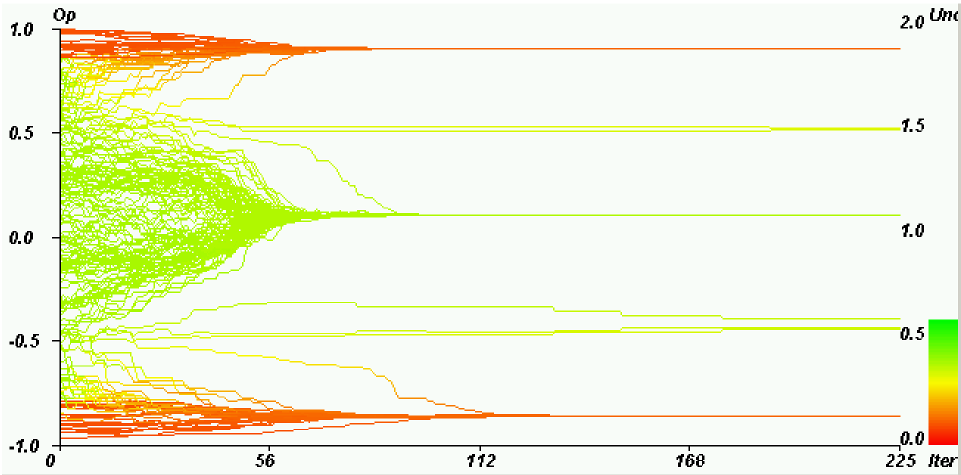


Figure 1: Example of central convergence. Horizontal axis: iterations. Vertical axis: opinions. Colored axis: uncertainties. $p_e = 0.2$, $U = 0.4$, $\mu = 0.5$, $\delta = 0$, $u_e = 0.1$, $N = 200$. The majority (96%) of the moderate agents (initially green, between the two extremes) are not attracted by the extremes.

Figure 2, obtained for $U = 1.2$ shows an example of bipolarization. In this case, the moderate agents are attracted by one of the extremes according to their initial position.

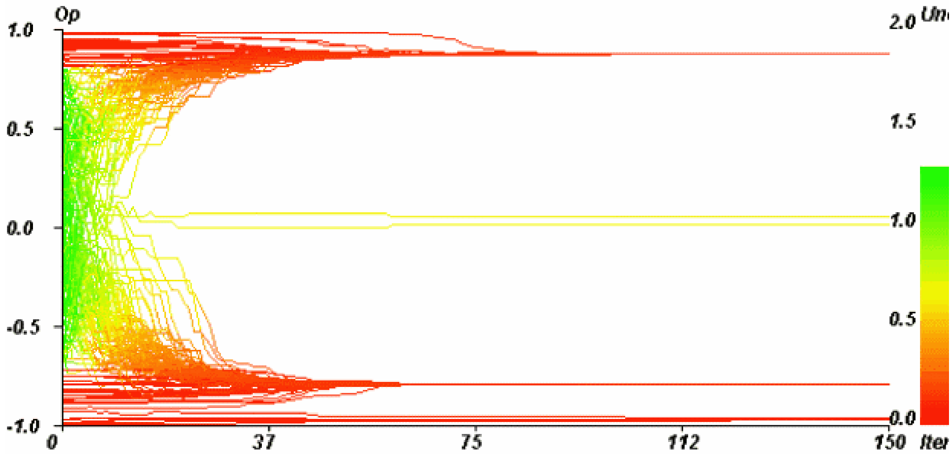


Figure 2: Example of both extremes convergence. Horizontal axis: iterations. Vertical axis: opinions. Colored axis: uncertainties. $p_e = 0.25$, $U = 1.2$, $\mu = 0.5$, $\delta = 0$, $u_e = 0.1$, $N = 200$. The initially moderate agents (initially green, between the two extremes) split and become extremists (43% on the positive side, 56% on the negative side).

Figure 3 obtained for $U = 1.4$ shows an example of single polarization. In this case, the majority of the population is attracted by one of the extremes. This behavior can take place even when the number of initial extremists is the same at both extremes.

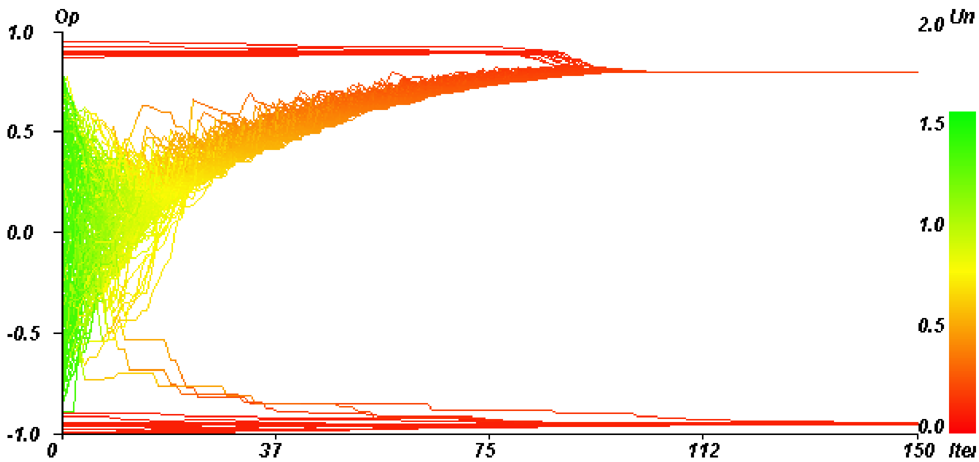


Figure 3: Example of single extreme convergence. Horizontal axis: iterations. Vertical axis: opinions. Colored axis: uncertainties. $p_e = 0.1$, $U = 1.4$, $\mu = 0.5$, $\delta = 0$, $u_e = 0.1$, $N = 200$. The majority (98.33%) of initially moderate agents (initially green, between the two extremes) is attracted by the negative extreme.

For another sample drawn from the same initial distribution, all other parameters being equals, one can even observe a central convergence (see figure 4).

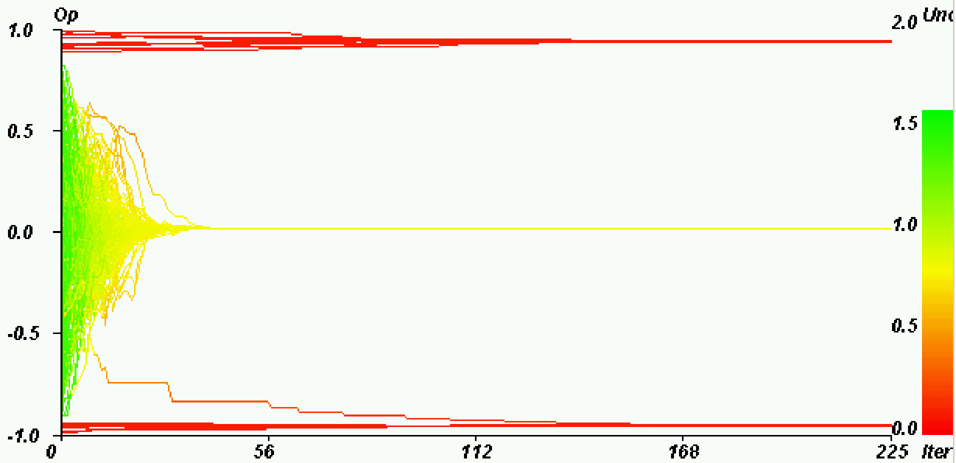


Figure 4: Central convergence for the same parameters as in figure 3. Horizontal axis: iterations. Vertical axis: opinions. Colored axis: uncertainties. $p_e = 0.1$, $U = 1.4$, $\mu = 0.5$, delta = 0, $u_e = 0.1$, $N = 200$. The majority stays at the center (Only one agent joins the negative extreme).

The instability of attractor is confirmed by the master equation analysis (Faure et al. 2002). For a perfect initial uniform distribution of opinions, the master equation (which is deterministic) displays a symmetric attractor (either central or on both extremes) in the region where single extreme convergence is obtained with the multi-agent RA model. But any slight asymmetry in the initial distribution changes the central convergence into a single sided extreme attractor (which side being determined by the asymmetry).

The color coding of the uncertainties shows that in all three cases clustering not only occurs among the opinions but also among uncertainties: e.g. when extremism prevails, it prevails in both opinions and uncertainties.

2.3.2. General results of the parameter space exploration

2.3.2.1. Convergence type indicator

We expressed the results of the exploration with an indicator of the convergence type, denoted y. To compute indicator y, we consider the population of opinions after convergence, and:

- We compute the proportions p'_+ and p'_- of the initially moderate agents which became extremists in the positive extreme or negative extreme.
- The indicator is then defined by: *(Eq. 8)* $y = p'^2_+ + p'^2_-$.

The value of this indicator indicates the type of convergence:

- If none of the moderate agents becomes extremist, then p'_+ and p'_- are null and $y = 0$.
- If half of the initially moderate converge to the positive extreme and half to the negative one, we have $p'_+ = 0.5$ and $p'_- = 0.5$, and therefore $y = 0.5$.
- If all the moderate agents go to only one extreme (say the positive one), we have $p'_+ = 1$ and $p'_- = 0$, and therefore $y = 1$.
- The intermediate values of the indicator correspond to intermediate situations.

2.3.2.2. Typical patterns of *y*

We found that the exploration of the model can be conveniently presented as variations of y with U and p_e. This representation leads to one typical pattern of average y for $\delta = 0$, and another one for $\delta > 0$. There is therefore a significant change between the cases where the proportion of positive and negative extremists is exactly the same, and when it is slightly different. When the other parameters u_e and μ are modified, the global shape of the patterns remains similar: only the position of the boundaries between the convergence zones varies.

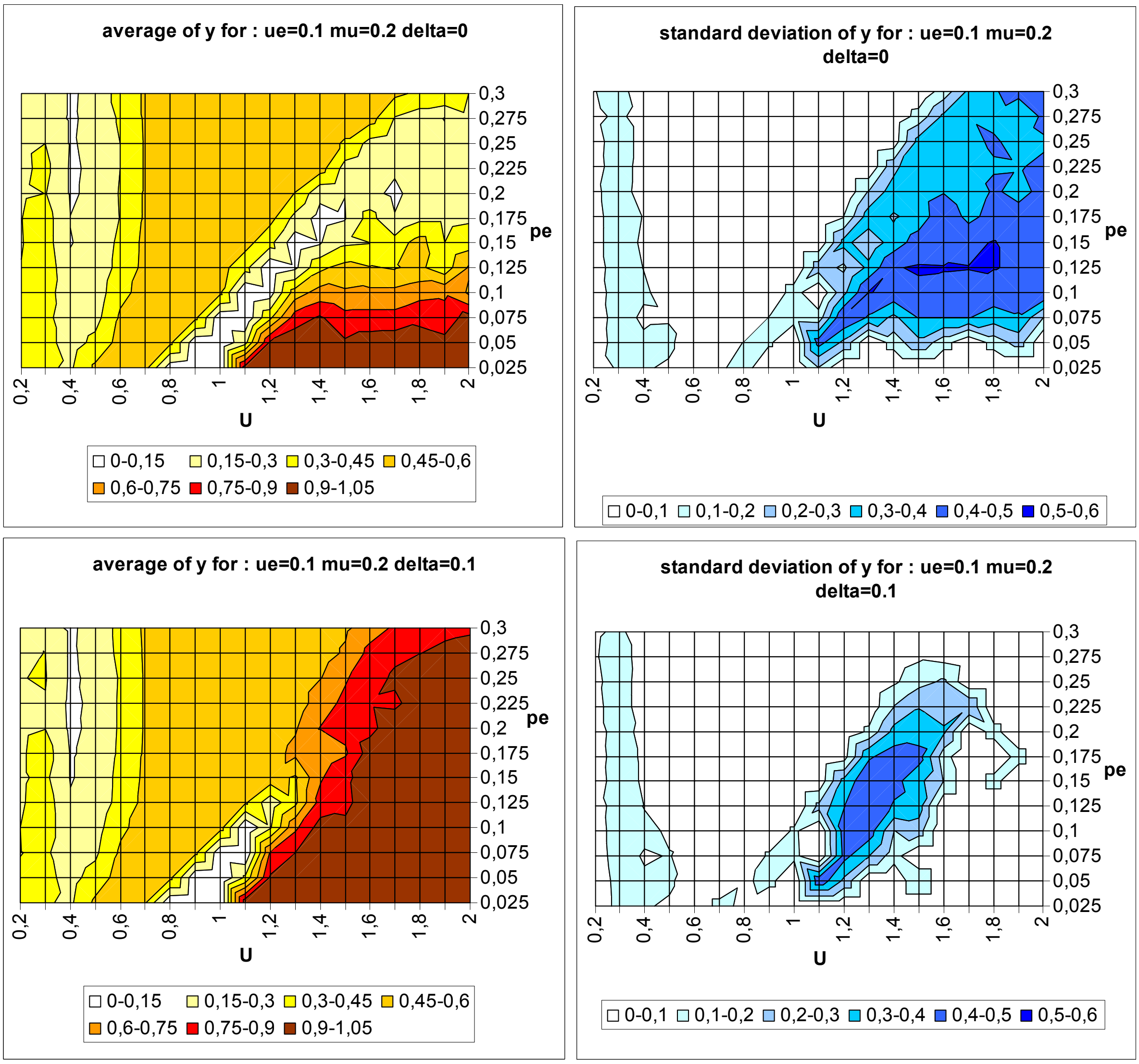


Figure 5: Typical pattern of average and standard deviation of indicator y (50 simulations at each point of the graph) as a function of the uncertainty of the moderate agents (U) and the global proportion of extremists (p_e) for $\delta = 0$ (top) and $\delta = 0.1$ (bottom). The other parameters are fixed: uncertainty of the extremists $u_e = 0.1$, intensity of interactions $\mu = 0.2$, initial relative difference between the extremists, $\delta = 0.1$. On the graph of average y, the yellow or white zones on the left part correspond to central convergence, the orange, typically in the upper middle part to both extremes, and brown at the bottom right to single extreme.

The typical patterns obtained for $\delta = 0$ and $\delta > 0$ are shown on figure 5. In this figure, each point of the grid corresponds to 50 simulations with 1000 agents. One can identify four regions with different average y values corresponding to the three dynamical regimes: two white–yellow zones (one on the left and the other one starting in diagonal from the lower middle part) corresponding to central convergence, one orange zone (drawing a triangle in the middle zone) for double extreme convergence and one brown zone (at the bottom right): single extreme convergence).

The dynamical regimes diagrams of figure 5 display large regions of intermediate *y* values: "pure" dynamical regimes, corresponding to $y = 0$ or 0.5 or 1, are separated by "crossover" regions where intermediate average *y* values and high standard deviation is due to a bimodal distribution of "pure" attractors depending on random sampling of initial conditions and pairing as confirmed by more detailed exploration (Deffuant et al., 2002).

3. Opinion dynamics on networks

3.1. Von Neumann neighborhood on grid

We then conduce simulations of this model changing the interaction structure from a complete graph, the totally connected case discussed above, to a regular grid (torus) with a Neumann neighborhood (connectivity $k = 4$).

To illustrate the dynamics change compare to the preceding case, we present firstly the cases taken for illustrating typical dynamics of the totally connected case (cf. Fig.1 to 4).

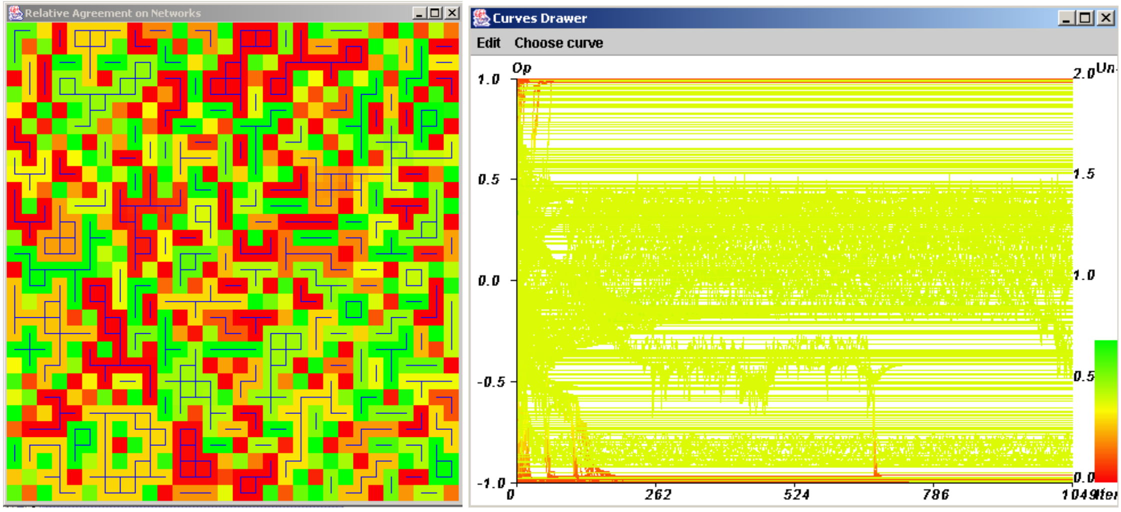


Figure 6: The parameters are similar to figure 1 are p_e=0.2, U=0.4, μ=0.5, δ=0, u_e = 0.1, in the totally connected case, it results in a central convergence. We can observe here that the system is highly clustered due to the social network taken. Colors code for opinion between –1 (red) and +1 (green). The represented blue links correspond to the relationships in the grid that verify the interaction condition (Amblard and Deffuant, 2003). The right curve corresponds to the opinion curve during simulation.

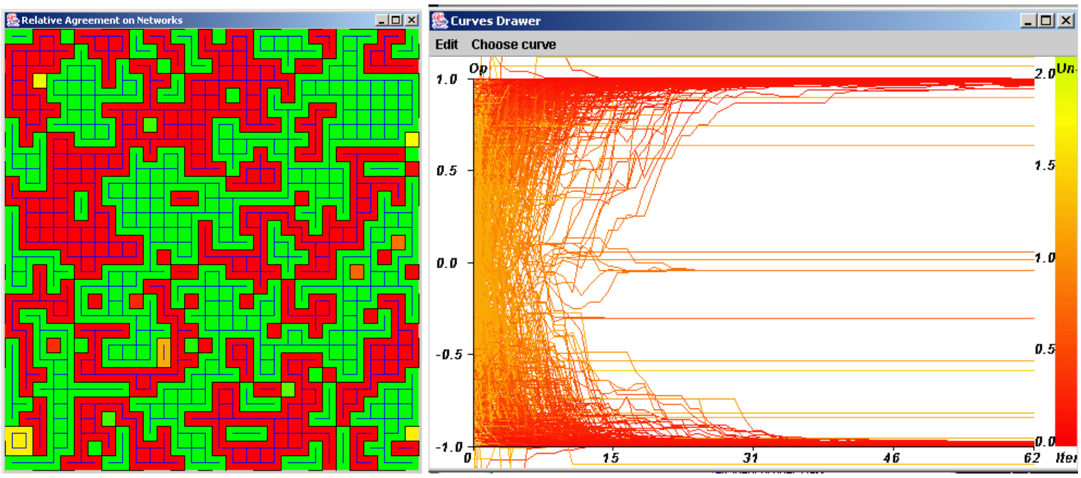


Figure 7: The parameters similar to the figure 2 are: p_e=0.25, U=1.2, μ=0.5, δ =0, u_e=0.1. We again (as in the figure 2) observe a both extremes convergence for the same value of parameters. The right curve represents the opinion evolution in time.

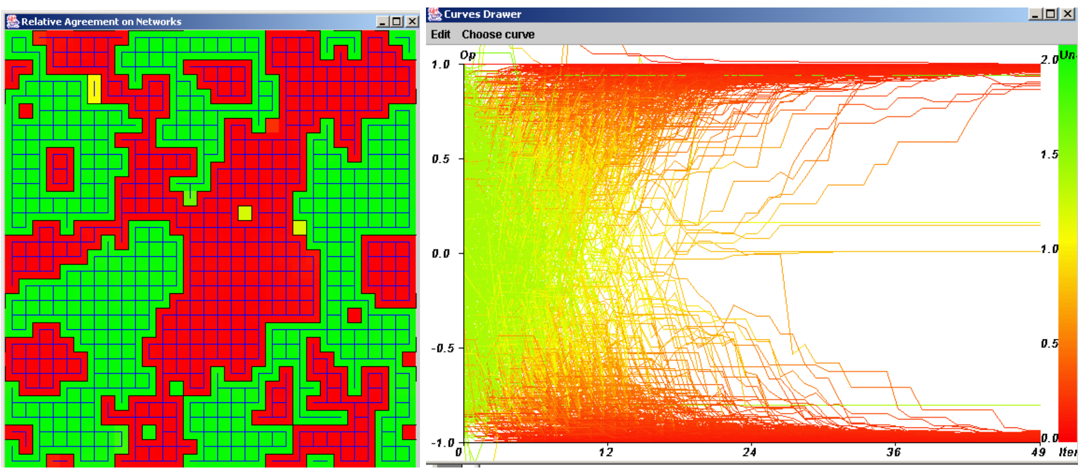


Figure 8: The parameters taken are similar to the ones of figure 3 and 4, p_e=0.1, U=1.4, μ=0.5, δ = 0, u_e=0.1. We obtain a both extremes convergence contrary to the figure 3 that corresponds to a single extreme convergence and to the figure 4 that corresponds to a central convergence.

Generalizing the exploration for many parameters values, we then only observe highly clustered case (see Figure 6) or both extremes convergence, we also observe cases of central convergence. But the main cases correspond to both extreme convergences. We do not observe single extreme convergence cases.
We then systematically explore the parameter space for *U* and p_e, as represented on figure 5 for the totally connected case (Figure 9).

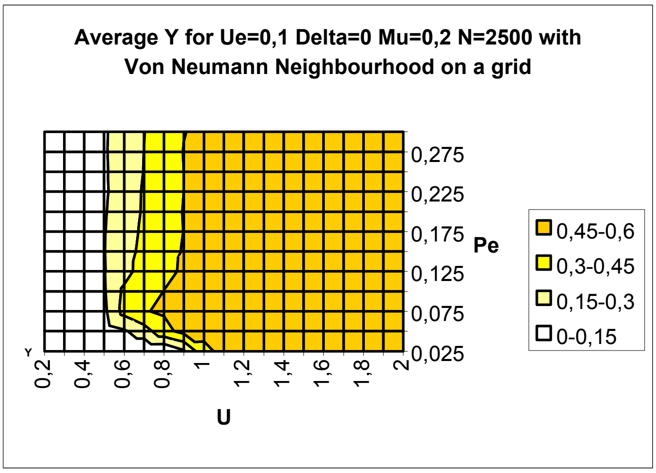


Figure 9: Exploration of the parameter space formed by U and p_e, other parameters are u_e=0.1 and μ=0.2, δ =0. The main observation is the absence of single extreme convergence (y never reaches values close to 1).

Concerning this exploration, the main observations we can do is on the one hand, the general decrease of *y* which never attain values over 0.6 illustrating the disappearance of the single extreme convergence case. On the other hand, we can observe on the left zone a zone of strong clustering that results in central convergence which let the place to both extreme convergence when we increase the uncertainty of the moderate.
The observations can be explained as follows:

- For small values of *U* a large number of clusters appear because the agents tend to be isolated: there is a high probability all their neighbors have too far opinions to be influential or to be influenced. This is also the case of the extremists, which are therefore not particularly influential for small *U*.
- For high values of *U*, the agents are on the contrary very likely to find interlocutors within their neighborhood. The influence of extremists propagates following the graph, first by attracting their own neighbors and then the others. The contamination is stopped when the formed cluster encounters another cluster of opposite opinion. Then the diffusion simply stops to invade the population.
- A possible explanation for the lack of single extreme convergence cases is this propagation which is prevented to occur when the connectivity is high, because the majority always attracts back the agents which are occasionally attracted by one extreme (leading to the phenomenon of global drift to one extreme, when the majority losses contact with the other extreme). With the Von Neumann neighborhood, for high values of *U*, each one of the extremes is able to disseminate within its neighborhood, which prevents the single extreme convergence to appear, except in very particular cases of extremists positioning within the networks or for high values of δ (difference between initial proportions of extremists at each extreme). When the initial number of extremist is the same at each extreme, we only observe a higher final proportion of one extreme (cf. Figure 10), but never the single extreme convergence observed in the fully connected case.

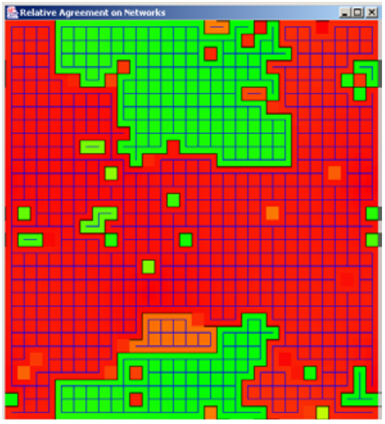


Figure 10: For the parameters U=0.8, u_e=0.1, p_e=0.01, δ =0, μ=0.2 we occasionally observe some final dominance of one extreme but never a single extreme convergence.

3.2. De Moore's neighborhood on grid

Exploring the model on a grid with a De Moore neighborhood (Figure 11), we observe that single extreme convergence does not appear either.

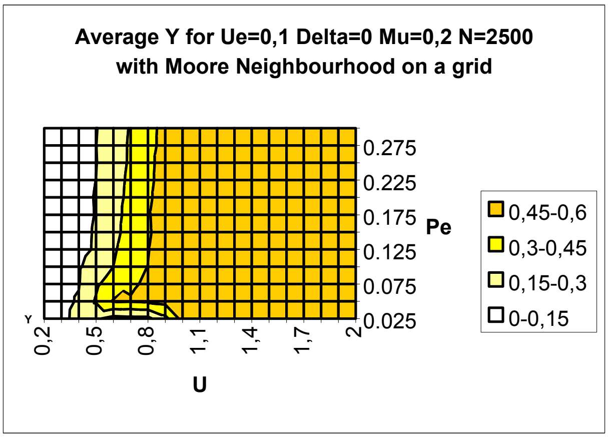


Figure 11: Systematic exploration with a De Moore neighborhood with u_e=0.1, δ =0 and μ=0.2. The single extreme convergence does not appear either.

If a De Moore neighborhood with an average connectivity of 8 is not sufficient to observe the single extreme convergence case, we have to increase this average connectivity. We then have to find a network model that enables to tune easily the average connectivity of the graph and also enables to go from regular networks to random ones because we want to test also the influence of the network regularity. Then we selected the *β*-model of small-worlds by (Watts, 1999) which satisfies these requirements.

3.3. Exploration on small-worlds

3.3.1. The Small World β-model (Watts, 1999)

Basically, starting from a regular structure (in our case a regular network over a circle) of connectivity *k*, we remove each link with the probability *β*, reconnecting it at random. In our case *k* has to be odd for us to take as a starting structure a circle with for each agent *k/2* relations from each side. The two parameters of the model are then the average connectivity *k* and the applied noise *β*.

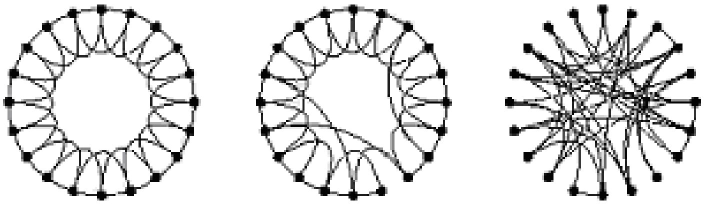
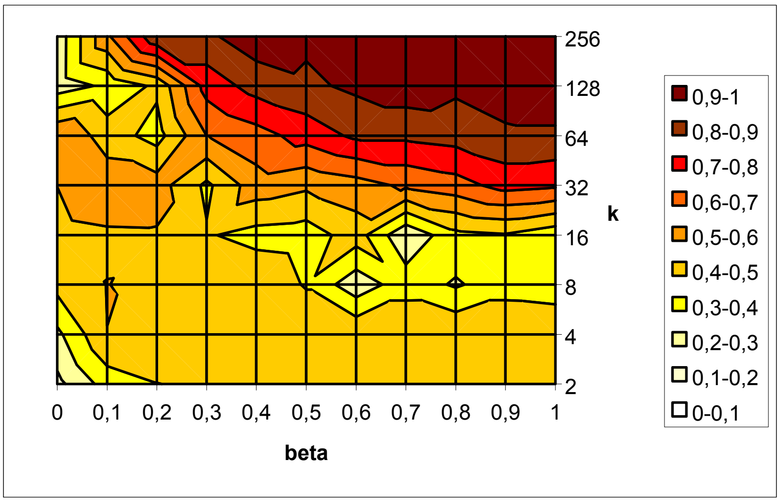


Figure 12: The β-model of (Watts, 1999) enables to go from regular graphs for low values of β (on the left) to random graphs for high values of β (on the right) using rewiring of edges.

3.3.2. The network effect for U=1.8 and p_e=0.05

We then selected a particular value of U and p_e that corresponds to a single extreme convergence in the fully connected case: a large uncertainty of the population U=1.8, and a low rate of extremist p_e=0.05 (bottom right part of the Figure 5). We run 50 replicas of the models on networks obtained when β ranges from 0 to 1, thus from regular networks to totally random ones and k ranges from 2 to 256 following the powers of 2 thus from 0.2% to 25% of the population (1000 individuals). Beyond the connectivity of 25%, the behavior of the model is the same as in the totally connected case, whatever the value of β.



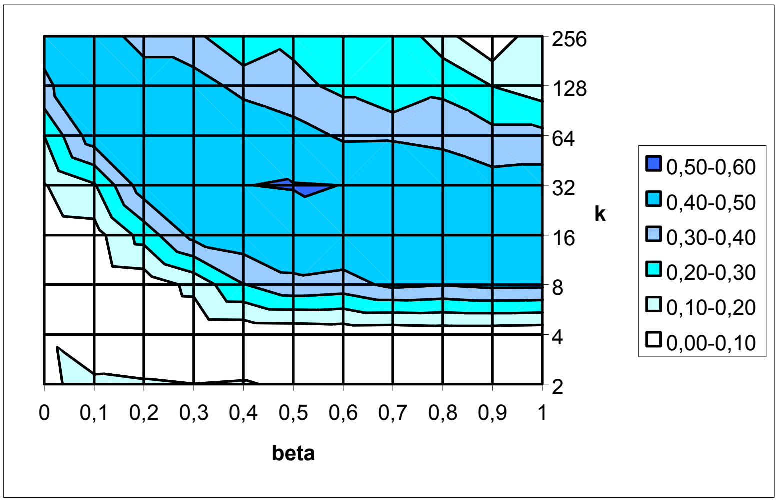


Figure 13: Exploration of the effect of a network following a small-world topology (β and k connectivity parameter) on the dynamics of the model for U=1.8, u_e=0.1, N=1000, μ=0.1, δ=0, p_e= 0.05 (single extreme convergence for the totally connected case)

We observe (see figure 13) a transition from the double extreme convergence to single extreme convergence case observed when the connectivity (k) increases. Within the transition zone, the high standard deviation of y corresponds to a mix between central convergence and single extreme convergence. The analysis of the traces of the opinions evolution for several simulations confirms the hypothesis expressed in section 3.1. For low connectivity each extremist influences its neighborhood which rapidly becomes extremist as well. We therefore obtain several clusters each one being in general controlled by one extremist, leading to a double extreme convergence. When the connectivity reaches a critical value, the population tends to regroup at the center which gives the possibility of a drift to one extreme. The central convergence cases take place when the central majority looses contact with both extremes. This situation is favored by low level of connectivity. When we increase the connectivity, this situation of a single extreme convergence regularly takes place.

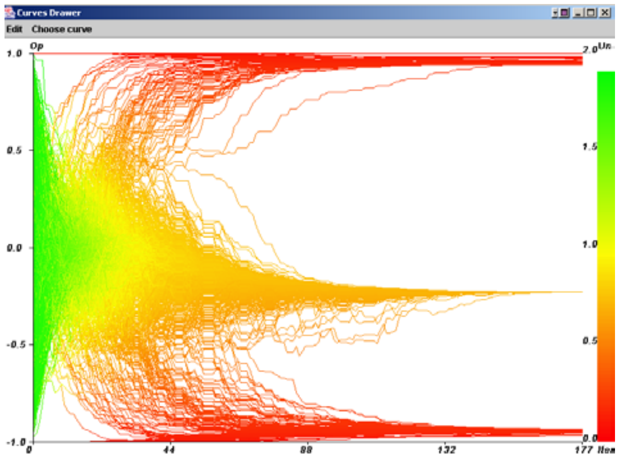


Figure 14: Trace of the opinion evolution for the preceding case with k=8 and β =0.8. We are in the zone of high standard deviation. The drift towards the negative extreme at the bottom does not take place because of a too low connectivity.

Moreover, the transition takes place for higher connectivity when beta decreases, i.e. when networks get more regular. Our hypothesis is that the regularity of the network reinforces the local effect which favors a fast local propagation of each initial extremist influence, leading to a double extreme convergence.

3.3.3. Influence of the network for other values of U

We selected three typical points, one for each convergence type with no mixing between different convergence cases in the totally connected case.

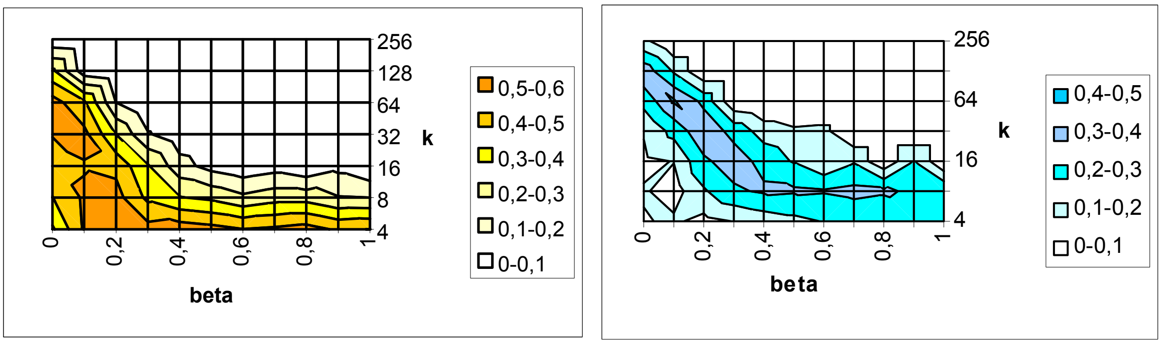


Figure 15: Simulations for U=1.0 and p_e=0.05 other parameters are taken constant considering the figure 13. This point corresponds in the totally connected case to a central convergence case as we can observe when we increase the connectivity.

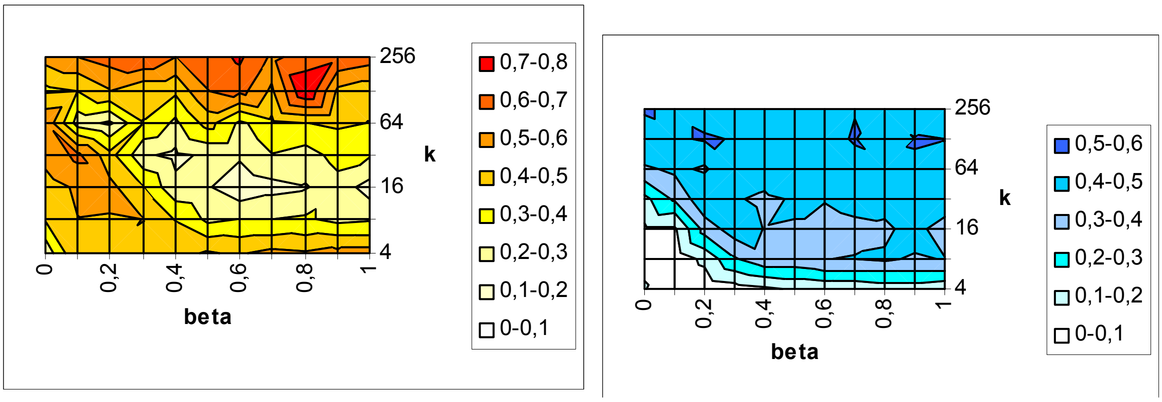


Figure 16: Simulations for U=1.2 and p_e=0.05 other parameters are taken constant considering the figure 13. This point corresponds in the totally connected case to a double extreme convergence case as we can observe when we increase the connectivity.

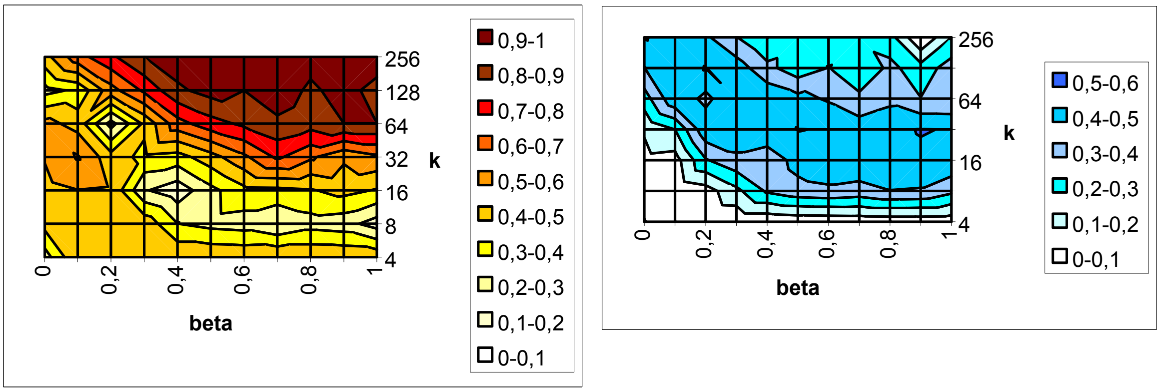


Figure 17: Simulations for U=1.4 and p_e=0.05 other parameters are taken constant considering the figure 13. This point corresponds in the totally connected case to a single extreme convergence case as we can observe when we increase the connectivity.

All the chosen points lead to quite similar dynamics when we make vary β and k. For small connectivity we observe a majority of double extreme convergences as for high values of connectivity we observe the convergence case observed in the totally connected case. Between those both case, there is a zone of mix between central convergence and the convergence case observed in the totally connected case. Moreover, the effect of the noise on the network leads also to the same property that is when we increase beta, we are then closer to random networks and the transition occurs sooner, for lower connectivity values.

In Figure 16, there is a transition zone between two zones of both extreme convergence. The two observed both extreme convergences are not exactly the same. For low connectivity it results mainly from the aggregation of local processes of convergence towards one extreme locally and for higher connectivity, it results from a global convergence of the central cluster, which is cut into two, each part being attracted by an extreme.

4.Discussion and perspectives

We proposed some results about the influence of different small world network structures on the model of relative agreement opinion dynamics, in presence of extremists. We found that there is a critical level of connectivity, which allows the single extreme convergence to take place. This critical level of connectivity increases when the regularity of the network increases. This result can be explained by the necessity of a first phase of global central clustering, for the single extreme convergence to take place. A low connectivity and high regularity of the network favor a fast local propagation of extremism, which prevents the global central clustering to take place.

Other studies have to be conducted to understand the role of other parameters of the model; for instance μ, the speed of convergence between two opinions which could play an important role, by speeding or slowing down the local propagation of extremism. First simulations let us think that the population size is also a critical parameter when the networks are regular. The critique of these results in a sociological perspective is also a major challenge which is out of the scope of this paper. However, the idea of the necessity of a critical level of connectivity and some disorder in the network for extreme opinions to invade a population does not seem counterintuitive, and could find rich interpretation in real life sociological phenomena.

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