



The role of network topology on extremism propagation with the relative agreement opinion dynamics

Frédéric Amblard*, Guillaume Deffuant

*Laboratoire d'Ingénierie des Systèmes Complexes Cemagref 24,
avenue des Landais 63172 Aubière Cedex, France*

Received 19 March 2004; received in revised form 23 April 2004

Available online 21 July 2004

Abstract

In Deffuant et al. (*J. Artif. Soc. Soc. Simulation* 5 (2002) 4), we proposed a simple model of opinion dynamics, which we used to simulate the influence of extremists in a population. Simulations were run without any specific interaction structure and varying the simulation parameters, we observed different attractors such as predominance of centrism or of extremism. We even observed in certain conditions, that the whole population drifts to one extreme of the opinion, even if initially there are an equal number of extremists at each extreme of the opinion axis. In the present paper, we study the influence of the social networks on the presence of such a dynamical behavior. In particular, we use small-world networks with variable connectivity and randomness of the connections. We find that the drift to a single extreme appears only beyond a critical level of connectivity, which decreases when the randomness increases.

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PACS: 87.23.Ge; 89.75.Hc

Keywords: Social dynamics; Opinion formation; Social networks

*Corresponding author.

E-mail address: frederic.amblard@cemagref.fr (F. Amblard).

1. Introduction

Most of the social influence literature, especially the contribution from social scientists, insists on the role of pre-existing social structures, such as social networks. In general, we lack empirical data about the topology of social network, although recent efforts lead to the proposal of different models such as small-worlds [1] and scale-free networks [2,3]. The purpose of this paper is to study the influence of the network structure on opinion dynamics. Most of the models of opinion formation in the literature are based on an Ising-like influence dynamics of binary or discrete opinions [4–6] taking as an assumption the influence of the group as the whole on the individual opinion. Another well-studied model is the one proposed by Sznajd and its numerous extensions [7]. The influence of network topology, especially of small-world networks, on such models have been studied [8,9]. In Ref. [10] we proposed an individual-based simulation model of continuous opinion dynamics, the relative agreement model (RA model). This model can be considered as a variant of the “bounded confidence” model [11–14] based on random pair-interaction in which an individual influences another one only if the distance between their opinions, considered as continuous, is below a given threshold. Related models with a vector of binary traits rather than continuous opinions can be found in the literature [13,15]. In the “classical” bounded confidence model, the influence varies linearly with the distance between the opinions, until the threshold where it suddenly goes to zero. The relative agreement model considers opinions segments, defined by the opinion and an uncertainty around this opinion. Both the opinion and the uncertainty are real numbers. The influence takes into account the overlap between the two opinions segments and avoids the discontinuity. Moreover, the more certain is the agent, the more convincing or influential it is. We model extremists in the population as individuals with a very low uncertainty and an opinion located at the extremes of the initial opinion distribution. When such extremists are initially present in the population, and when each individual is connected to all the others, the simulations exhibit three types of convergence: the “central convergence” in which the extremists attract only a very limited part of the population, which is initially close to the extremes; the “both extreme convergence” in which the population splits into two groups, each converging to one extreme; the “single extreme convergence” in which the whole population converges to one of the extremes [10]. The last type of convergence was the most unexpected. It appears even when the initial number of extremists at each extreme is the same. In this paper, we study the influence of some social network structures on the occurrence of the single extreme convergence.

In the first paragraph we briefly present the relative agreement model with extremists and the main results for the fully connected case. Readers who wanted to know more about this model can refer to Ref. [10]. We then consider social networks defined on a regular lattice with a Moore neighborhood. The single extreme convergence does not occur with these social networks. We then explore the behavior of the model on small-world networks [1] that enable us to tune both the average connectivity of the network and the randomness of its connections. The simulations

show that there is a critical level of connectivity beyond which the drift to a single extreme appears. We then discuss the observed behavior of our model.

2. The relative agreement model

2.1. Definition

We consider a population of N individuals. Each individual i is characterized by two variables, its opinion x_i and its uncertainty u_i , both being real numbers. We call segment $s_i = [x_i - u_i, x_i + u_i]$ of the opinion axis the opinion segment of individual i . In the following, we draw the opinions from a uniform distribution between -1 and $+1$. We suppose that random pair interactions take place among the individuals, during which they may influence each other’s opinion segment. In the simplest model of “bounded confidence” another individual j can influence agent i whenever its opinion is inside segment s_i . Here we use the relative agreement model which results in a continuous variation of the influence on the opinion axis and where the intensity of the influence depends on the agent j uncertainty u_j . More precisely, let us consider opinion segments $s_i = [x_i - u_i, x_i + u_i]$ and $s_j = [x_j - u_j, x_j + u_j]$. We define the agreement of agent i with j (it is not symmetric) as the overlap of s_i and s_j , minus the non-overlapping part. The overlap h_{ij} is given by

$$h_{ij} = \min(x_i + u_i, x_j + u_j) - \max(x_i - u_i, x_j - u_j). \tag{1}$$

The non-overlapping width is

$$2u_i - h_{ij}. \tag{2}$$

The agreement is the overlap minus the non-overlap

$$h_{ij} - (2u_i - h_{ij}) = 2(h_{ij} - u_i). \tag{3}$$

The relative agreement is the agreement divided by the length of segment s_i

$$\frac{2(h_{ij} - u_i)}{2u_i} = \frac{h_{ij}}{u_i} - 1. \tag{4}$$

If $h_{ij} > u_i$, then the modifications of x_j and u_j by the interaction with i are multiplied by the relative agreement:

$$x_j := x_j + \mu \left(\frac{h_{ij}}{u_i} - 1 \right) (x_i - x_j), \tag{5}$$

$$u_j := u_j + \mu \left(\frac{h_{ij}}{u_i} - 1 \right) (u_i - u_j), \tag{6}$$

where μ is a rate of the dynamics. If $h_{ij} \leq u_i$, there is no influence of i on j . The main features of the relative agreement model are:

- During interactions, agents not only influence each other’s opinions but also each other’s uncertainties.

- The influence is not symmetric when the agents have different uncertainties; “confident” agents (low uncertainty) are more influential.

The influence (the modifications of x_j and u_j) varies continuously when x_j, u_j, x_i and u_i vary continuously. Then, as a reminder, the only stochastic part of this model concerns the selection of the pair-interactions that are drawn at random.

2.2. Addition of extremists

We now introduce extremists into our population: we suppose that these individuals have opinions located at the extremes of the opinion distribution are more confident than the non-extremist (moderate) individuals. We define therefore two values, on the one hand, their initial uncertainty: u_e the uncertainty of all the extremists, and on the other hand, U the initial uncertainty of the moderate, supposed higher than u_e . We define also p_e as the global proportion of extremists in the population. p_+ and p_- are then the proportion of extremists at the positive or negative extreme opinions. The relative difference between the proportion of positive and negative extremists is noted δ :

$$\delta = \frac{|p_+ - p_-|}{p_+ + p_-}. \quad (7)$$

In practice, we first randomly draw opinions of $(1 - p_e)N$ agents of the population from a uniform distribution between -1 and $+1$. Then we initialize Np_+ agents to $+1$ and Np_- most negative opinions to -1 , moreover we initialize them with the uncertainty u_e , and the others with the uncertainty U .

2.3. Results on the totally connected case

2.3.1. Three convergence types

From Ref. [10], this very simplified model of extremism, exhibits three different dynamical regimes, depending on the parameters: the “central convergence” in which the extremists have a small influence on the rest of the population; the “both extremes convergence” in which the whole population becomes extremist with an almost equal number of extremists at each extremes; and the “single extreme convergence” where the whole population drifts to one single extreme. The following set of figures, obtained from numerical simulations, exhibits the three different dynamical regimes. The x -axis codes for time (number of iterations), the y -axis for opinions, and the level of gray for uncertainty. Each trajectory allows following the evolution in opinion and uncertainty of one individual agent. Common parameters are, $\mu = 0.5, \delta = 0, u_e = 0.1, N = 200$. The uncertainty parameter U of the general population is increased from Fig. 1(a) to Fig. 1(d). Fig. 1(a), obtained for $U = 0.4$, shows an example of central convergence. In this case, only a marginal part of the initially non-extremists became extremist (4%). Fig. 1(b), obtained for $U = 1.2$ shows an example of bipolarization. In this case, the moderate agents are attracted by one of the extremes according to their initial position. Fig. 1(c) obtained for

$U = 1.4$ shows an example of single polarization. In this case, the majority of the population is attracted by one of the extremes. This behavior can take place even when the number of initial extremists is the same at both extremes. But for another sample drawn from the same initial distribution, all other parameters being equal, one can even observe a central convergence (see Fig. 1(d)). The sensitivity to initial conditions and to random sampling is a general feature of the model in transition regions.

The instability of the attractor is confirmed by the master equation analysis [16]. For a perfect initial uniform distribution of opinions, the master equation (which is deterministic) displays a symmetric attractor (either central or on both extremes) in the region where single extreme convergence is obtained with the multi-agent RA model. But any slight asymmetry in the initial distribution changes the central convergence into a single sided extreme attractor (which side being determined by the asymmetry). The coding of the uncertainties shows that in all three cases clustering not only occurs among the opinions but also among uncertainties: e.g. when extremism prevails, it prevails in both opinions and uncertainties.

2.3.2. General results of the parameter space exploration

Convergence type indicator. We expressed the results of the exploration with an indicator of the convergence type, denoted y . To compute indicator y , we consider the population of opinions after convergence, and:

- We compute the proportions p'_+ and p'_- of the initially moderate agents which became extremists in the positive extreme or negative extreme.
- The indicator is then defined by:

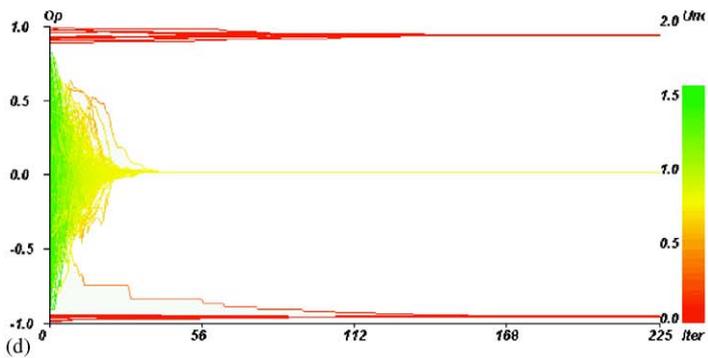
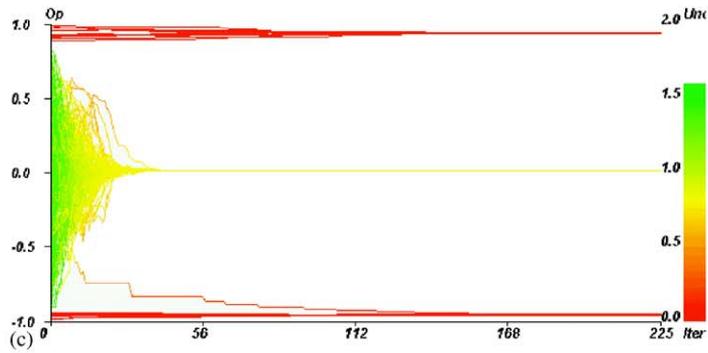
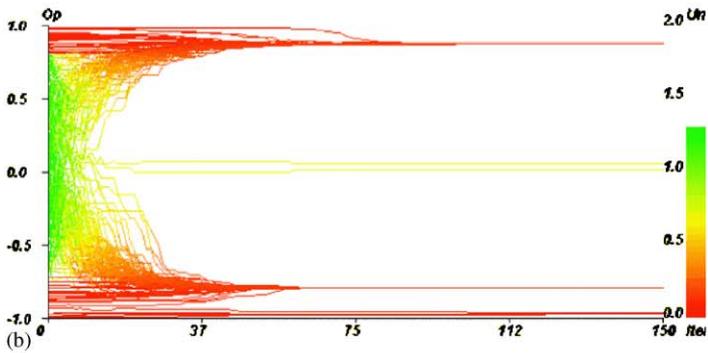
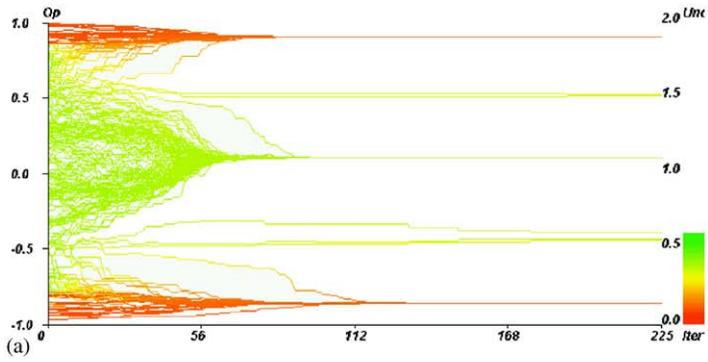
$$y = p'_+{}^2 + p'_-{}^2. \quad (8)$$

The value of this indicator indicates the type of convergence:

- If none of the moderate agents becomes extremist, then p'_+ and p'_- are null and $y = 0$.
- If half of the initially moderate converge to the positive extreme and half to the negative one, we have $p'_+ = 0.5$ and $p'_- = 0.5$, and therefore $y = 0.5$.
- If all the moderate agents go to only one extreme (say the positive one), we have $p'_+ = 1$ and $p'_- = 0$, and therefore $y = 1$.
- The intermediate values of the indicator correspond to intermediate situations.

Typical patterns of y . We found that the exploration of the model can be conveniently presented as variations of y with U and p_e . This representation leads to one typical pattern of average y for $\delta = 0$, and another one for $\delta > 0$. There is therefore a significant change between the cases where the proportion of positive and negative extremists is exactly the same, and when it is slightly different. When the other parameters u_e and μ are modified, the global shape of the patterns remains similar, only the position of the boundaries between the convergence zones varies.

The typical patterns obtained for $\delta = 0$ and $\delta > 0$ are shown on Fig. 2. In this figure, each point of the grid corresponds to 50 simulations with 1000 agents. One



can identify four regions with different average y values corresponding to the three dynamical regimes: two white zones (one on the left and the other one starting in diagonal from the lower middle part) corresponding to central convergence, one gray zone (drawing a triangle in the middle zone) for double extreme convergence and one dark gray zone (at the bottom right) for single extreme convergence. The dynamical regimes diagrams of Fig. 2 display large regions of intermediate y values: “pure” dynamical regimes, corresponding to $y = 0$ or 0.5 or 1 , are separated by “crossover” regions where intermediate average y values and high standard deviation is due to a bimodal distribution of “pure” attractors depending on random sampling of initial conditions and pairing as confirmed by more detailed exploration [10].

3. Opinion dynamics on networks

3.1. Social network as a regular grid

Instead of a totally connected social network, we now run the model on a regular lattice (a torus) with a Moore neighborhood (connectivity $k = 8$). We systematically explore the parameter space for the uncertainty of the moderate individuals U and the proportion of extremists p_e (see Fig. 3).

We note that:

- y is always below 0.6 which shows that the single extreme convergence never occurs. We only observed single extreme convergence in very particular cases of extremists positioning or for high values of δ (difference between initial proportions of extremists at each extreme). When the initial number of extremist is the same at each extreme, we only observe a higher final proportion of one extreme, but never the single extreme convergence observed in the fully connected case.
- The transition from central to both extreme convergences has a shape similar to the one observed when the population is fully connected.

These observations can be explained as follows:

- For small values of U a large number of clusters appear because the agents tend to be isolated: there is a high probability all their neighbors have too far opinions to

Fig. 1. (a) Example of central convergence. Horizontal axis: iterations. Vertical axis: opinions. Colored axis: uncertainties. $p_e = 0.2$, $U = 0.4$, $\mu = 0.5$, $\delta = 0$, $u_e = 0.1$, $N = 200$. The majority (96%) of the moderate agents are not attracted by the extremes. (b) Example of both extremes convergence. $p_e = 0.25$, $U = 1.2$, $\mu = 0.5$, $\delta = 0$, $u_e = 0.1$, $N = 200$. The initially moderate agents split and become extremists (43% on the positive side, 56% on the negative side). (c) Example of single extreme convergence. $p_e = 0.1$, $U = 1.4$, $\mu = 0.5$, $\delta = 0$, $u_e = 0.1$, $N = 200$. The majority (98.33%) of initially moderate agents is attracted by the negative extreme. (d) Central convergence for the same parameters as in (c), the majority stays at the center (Only one agent joins the negative extreme).

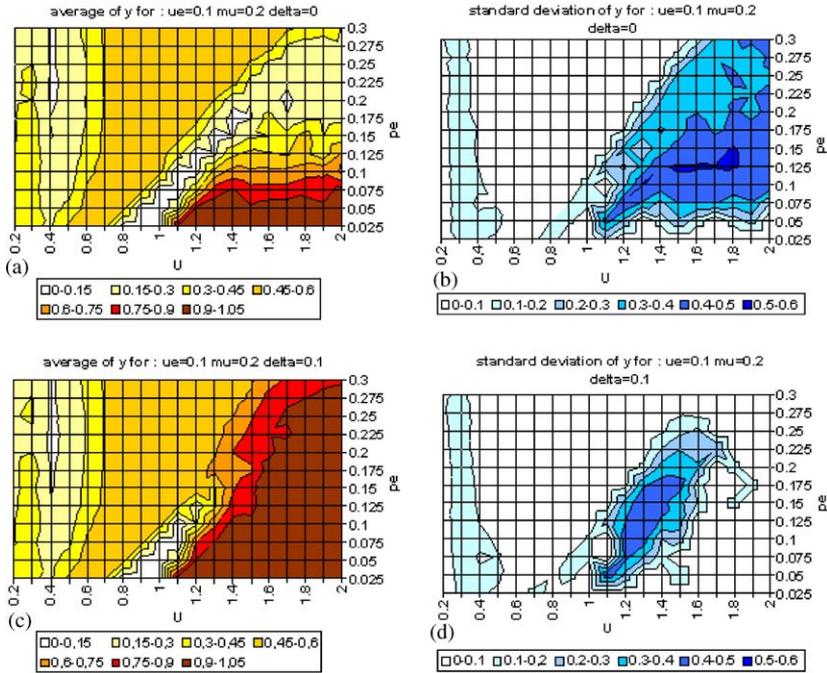


Fig. 2. Typical pattern of average and standard deviation of indicator y (50 simulations at each point of the graph) as a function of the uncertainty of the moderate agents (U) and the global proportion of extremists (p_e) for $\delta = 0$ (a), (b) and $\delta = 0.1$ (c), (d). The other parameters are fixed: uncertainty of the extremists $u_e = 0.1$, intensity of interactions $\mu = 0.2$, initial relative difference between the extremists, $\delta = 0.1$. On the graph of average y , the yellow or white zones on the left part correspond to central convergence, the orange, typically in the upper middle part to both extremes, and brown at the bottom right to single extreme.

- be influential or to be influenced. This is also the case of the extremists, which are therefore not particularly influential for small U .
- For high values of U , the agents are on the contrary very likely to find interlocutors within their neighborhood. The influence of extremists propagates following the graph, first by attracting their own neighbors and then the others. The contamination is stopped when the formed cluster encounters another cluster of opposite opinion. Then the diffusion simply stops to invade the population.
 - A possible explanation for the lack of single extreme convergence is due to this local propagation of extremism. It does not occur when the connectivity is high, because the majority always attracts back the agents which are occasionally attracted by one extreme (leading to the phenomenon of global drift to one extreme, when the majority losses contact with the other extreme). With the Moore neighborhood, for high values of U , each extremist disseminates very rapidly within its neighborhood, which prevents the single extreme convergence to appear.

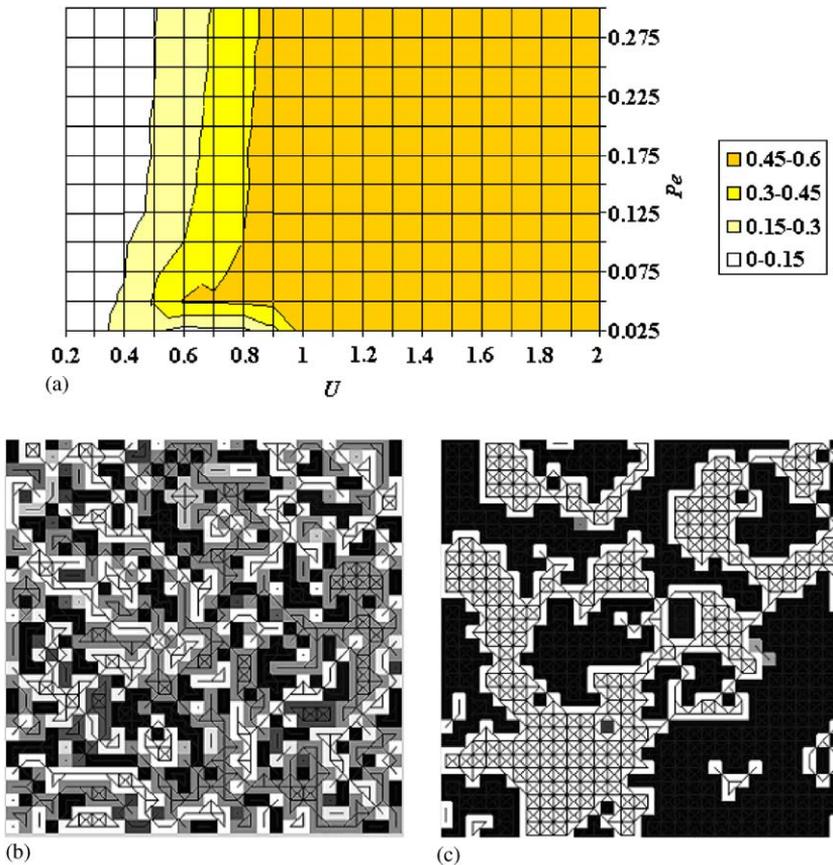


Fig. 3. (a) Exploration of the parameter space on a grid formed by U and p_e , with a Moore's neighborhood (connectivity 8), other parameters are $u_e = 0.1$ and $\mu = 0.2, \delta = 0$. The associated standard deviation is relatively low except for the zone corresponding to the transition for low values of p_e . The main observations are the general decrease of the y values and the absence of single extreme convergence (y never reaches values close to 1). (b) Final state of the system corresponding to the left part of a) for $U = 0.4$ and $p_e = 0.2$, the system is highly clustered. (c) Final state corresponding to the right part for $U = 1.4$ and $p_e = 0.2$, the system, even converging towards a double extreme, is much more cohesive.

If a De Moore neighborhood with an average connectivity of eight is not sufficient to observe the single extreme convergence case, we have to increase this average connectivity. We then need a network model that enables to tune easily the average connectivity of the graph and also the network regularity. We selected the β -model of small-worlds by [1] which satisfies these requirements.

3.2. Exploration on small-worlds

The β -model of small-world network is ruled by two parameters: the average connectivity k and the randomness of the connections p . Starting from a regular

structure (in our case a regular network over a circle) of connectivity k , we remove each link with the probability p , reconnecting it at random. In our case k has to be odd because each individual on the circle has $k/2$ connections in each side. We then studied the behavior of the model for a large uncertainty of the population $U = 1.8$, and a low rate of extremist $p_e = 0.05$ (bottom right part of the Fig. 2), leading to a single extreme convergence when the population is fully connected. We run 50 replications of the simulations on networks obtained when p ranges from 0 to 1, thus from regular networks to totally random ones and k ranges from 2 to 256 following the powers of 2 thus from 0.2% to 25% of the population (1000 individuals). Beyond the connectivity of 25%, the behavior of the model is the same as in the totally connected case, whatever the value of p .

We observe (see Fig. 4) a transition from double extreme convergence to single extreme convergence case when the connectivity (k) increases. Within the transition zone, the high standard deviation of y indicates that sometimes a central convergence and single extreme convergence. The analysis of the traces of the opinions evolution for several simulations confirms the hypothesis expressed in Section 3.1. For low connectivity each extremist influences its neighborhood which rapidly becomes extremist as well. We therefore obtain several clusters each one being in general, controlled by one extremist, leading to a double extreme convergence. When the

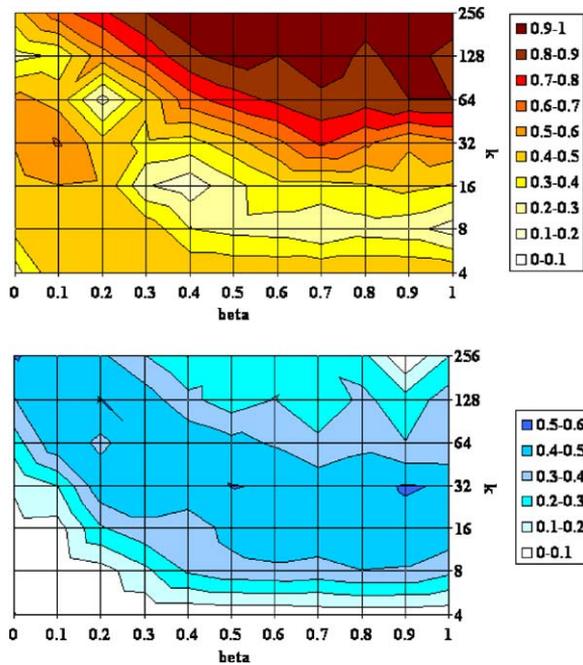


Fig. 4. Exploration of the effect of a network following a small-world topology with a ring substrate (β and k connectivity parameter) on the dynamics of the model for $U = 1.8$, $u_e = 0.1$, $N = 1000$, $\mu = 0.1$, $\delta = 0$, $p_e = 0.05$ (single extreme convergence for the totally connected case).

connectivity reaches a critical value, the population tends to regroup at the center and the fluctuations of this opinion cluster may lead to disconnect it from one extreme, leading to a drift to the other extreme. The central convergence takes place when the central majority loses contact with both extremes. This situation is favored by a low level of connectivity. When we increase the connectivity, this situation of a single extreme convergence regularly takes place as confirmed by the analysis of the distribution of convergence cases for a fixed value of p (cf. Fig. 5).

Moreover, the transition takes place for higher connectivity when p decreases, i.e., when networks get more regular. Our hypothesis is that the regularity of the network reinforces the local effect which favors a fast local propagation of each initial extremist influence, leading to a double extreme convergence. These results are robust considering the chosen substrate to build the network as for a square lattice Fig. 6 exhibits the same phenomenon.

Then, we selected three typical couples (U, p_e) corresponding to different convergence types when the population is fully connected.

All the chosen couples lead to quite similar dynamics when p and k vary. For small connectivity we observe a majority of double extreme convergences as for high connectivity we observe the convergence case observed in the totally connected case. We observe the systematic occurrence of central convergence in the transition between these behaviors. Moreover, when we increase p , we are closer to random networks and the transition occurs for lower connectivity values. In Fig. 7(b), there is a transition zone between two zones of both extreme convergences. The two observed both extreme convergences are due to different processes. For low

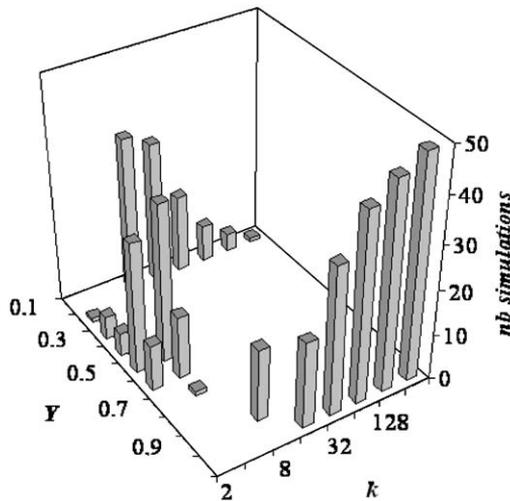


Fig. 5. Distribution of y for a small-world topology with k connectivity parameter on a logarithmic scale, for $\beta = 0.8, U = 1.8, u_e = 0.1, N = 1000, \mu = 0.1, \delta = 0, p_e = 0.05$. We observe the phase transition that occurs for values of connectivity around 8 and from that point that the mixing between single convergence case and central convergence tends to become uniformly single extreme convergence.

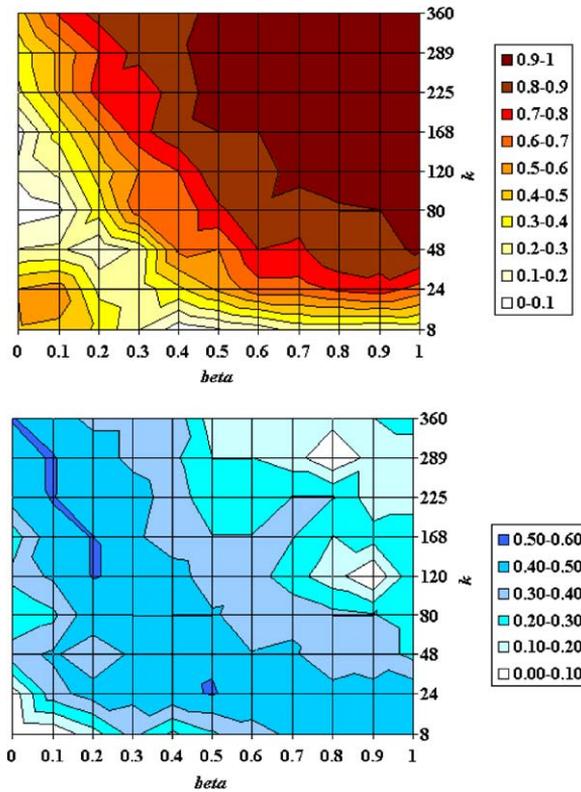


Fig. 6. Exploration of the effect of a network following a small-world topology with a grid substrate applying a generalized Moore neighborhood (β and k connectivity parameter) on the dynamics of the model for $U = 1.8$, $u_e = 0.1$, $N = 1000$, $\mu = 0.1$, $\delta = 0$, $p_e = 0.05$ (single extreme convergence for the totally connected case). Qualitative results are the same compared to a ring substrate even distortions are observed due to possible connectivity values to be tested.

connectivity it results mainly from the aggregation of local processes of convergence towards one extreme locally and for higher connectivity, it results from a global convergence of the central cluster, which is cut into two, each part being attracted by an extreme.

4. Discussion and perspectives

We run the relative agreement model with extremists on small-world social networks, with varying connectivity and randomness. We found that there is a critical level of connectivity, which allows the single extreme convergence to take place. This critical level of connectivity increases when the regularity of the network increases. This result can be explained by the need of a first phase of global central clustering, to allow the single extreme convergence to take place. A low connectivity

and high regularity of the network favor a fast local propagation of extremism, which prevents the global central clustering to take place. Comparing it to existing results [8] we do not observe a classical small-world effect for networks that are at the same time highly clustered and that have a low diameter. But we found for our

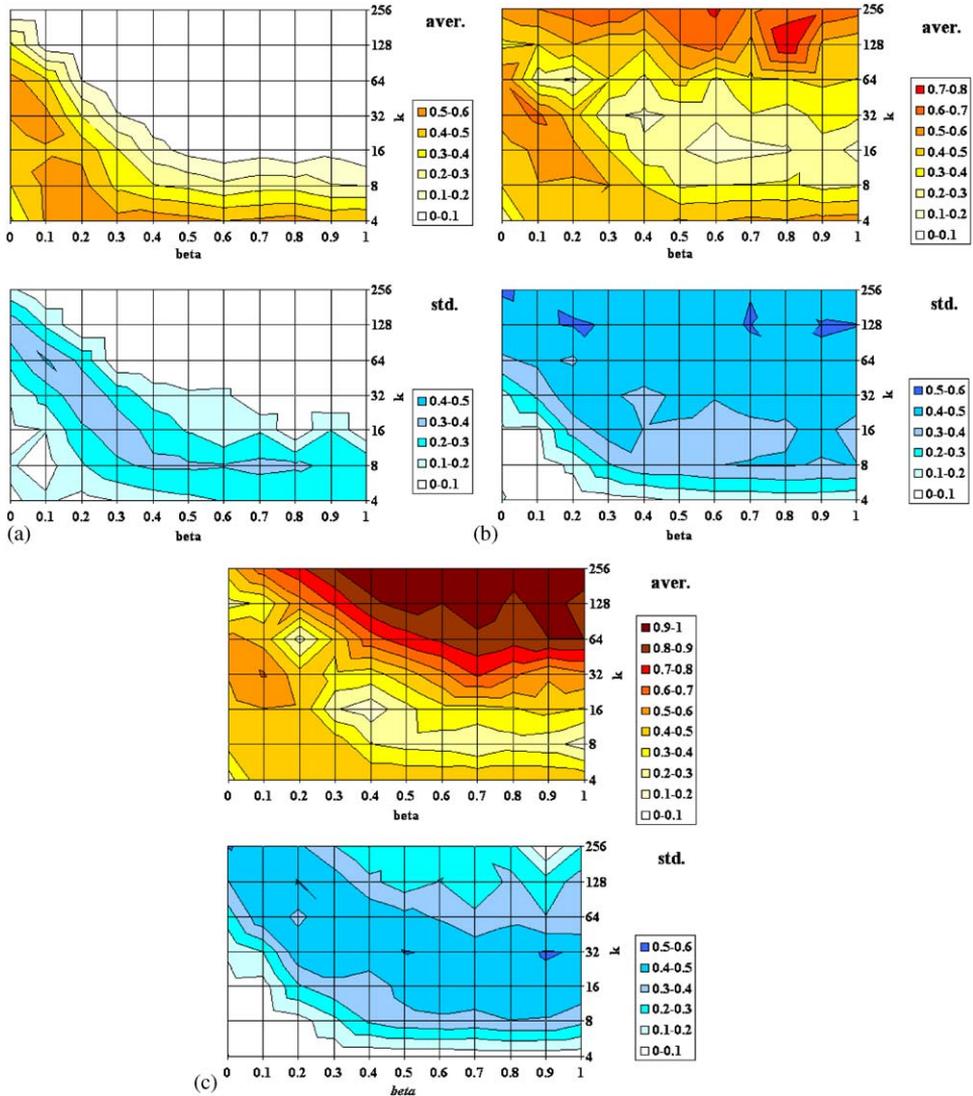


Fig. 7. Representation of the average of y over 50 replications at each point on the left and standard deviations on the right for different couples (U, p_e) other parameters are taken constant considering the Fig. 4. The three points taken corresponds in the totally connected case to (a) central convergence case ($U = 1.0$ and $p_e = 0.05$) (b) double extreme convergence case ($U = 1.2$ and $p_e = 0.05$) and (c) single extreme convergence case ($U = 1.4$ and $p_e = 0.05$) as we can observe when we increase the connectivity.

model that two phase transitions was observed in parameter zones that are usually kept unexplored, especially for high average degrees. Other studies have to be conducted to understand the role of other parameters of the model. For first simulations let us think that the population size is a critical parameter when the networks are regular. The critique of these results in a sociological perspective is also a major challenge which is out of the scope of this paper. However, the idea of the necessity of a critical level of connectivity and some disorder in the network for extreme opinions to invade a population does not seem counterintuitive, and could find rich interpretation in real life sociological phenomena. Moreover, several extensions of the model are under consideration. Among them, the addition of an opinion radicalization dynamics that could result in a bigger gap between opinions after the interaction, is currently studied with a socio-psychologist.

Acknowledgements

The authors are grateful to Gérard Weisbuch and to the anonymous referees for their very valuable comments on this paper. The work has been partially funded by the Conseil Régional d’Auvergne.

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