

# FreqNet: a new approach in formalizing social networks

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**Abstract:** This paper introduces FreqNet as a new way to formalise dynamic relationships, allowing social networks to emerge. The main idea of the proposal is that the probability of interaction associated to the links depends on an underlying opinion dynamics model on two dimensions. First experiments are presented and discussed using as a hypothesis of homophily considering that the probability of interaction depends both on a similarity in interest and in opinions.

**Keywords:** social simulation, agent based simulation, social networks.

## 1: Introduction

Artificial societies grow in popularity in studying the dynamics of group behaviour. Market dynamics, crowd behaviour, conflicts and many other topics relate to the many interactions between individual people which may sometimes result in the remarkable and sometimes devastating consequences. Hence the interest for the formalisation led physicists to develop the concept of 'social atoms', describing fixed networks of local interacting agents leading towards macro-level phenomena [1]. In response to the empirical evidence that social contacts between people are not only manifest in a local grid setting, but also include distant or weak links [2], [3] formalised small world networks, where local contacts were rewired by an arbitrary chance to a random contact. A next step in the formalisation of social networks was provoked by the scale-free property of complex networks, where the distribution of number of links per node or per agent follows a power law [4]. It leads to the proposal of preferential attachment process [5] and to the introduction of limits to the number of connections [5] or to aging processes for the links [9]. Whereas previous approaches basically assume the presence or absence of a connection in binary terms, in this paper we propose a description of connection in terms of frequency of interaction, this frequency being formalized as a probability. People and the networks they take part in are always changing, sometimes at a slow pace, where once close contacts end in long lingering contacts. Remember this old friend you met again last week and you didn't meet for 20 years, (by the way you forgot to mention him to the social network survey), indeed you didn't act as you never met before and you spent a great moment talking about the past but also about his company and the vacant position for which he was searching for someone. Basically the deletion of links as

presented in several models does not exist, the frequency of interaction decreases but the link can be “re”-activated at any moment.

The focus of this paper will be on the formalisation of a frequency-of-contact-based-network, abbreviated as FreqNet, and a formalisation of the attraction and repulsion forces that determine changes in contact frequency. Whereas the previously described networks all more-or-less impose a specific network structure on the agents, the FreqNet formalisation allows to formulate easily a dependence between agents’ attitudes and their probabilities of interaction (which in return influence at a meso-scale the opinion dynamics), thus allowing for the formation of evolving networks. This opens the possibility of studying the evolution of network structures as a function of interaction processes.

## 2: A formalisation of FreqNet

A critical element in FreqNet is the rationale behind the interaction between people. Let’s consider a population of  $n$  agents. In daily life the frequency on interaction is determined by a multitude of factors, such as (changes in) location, shared interests, family ties and many more. One key-driver identified in many psychological and behavioural studies is the similarity of people [6]. This similarity is based on several dimensions, e.g., work, sports, and politics. In our formalisation we start with 2 attitude dimensions: for a given agent  $j$ ,  $A_j$  and  $B_j$ , ranging from -1 to 1. People may attach different importance to these attitudes. Hence agent  $j$  will weight the dimensions with a  $\beta_j$  value:  $\beta_j * A_j$  and  $(1 - \beta_j) * B_j$ , considering that the total interest is equal to 1. We then formalised agent  $j$ ’s perceived dissimilarity with agent  $k$  as:

$$\Delta_{jk} = \Delta_{kj} = \frac{(\beta_j + \beta_k)}{2} * |A_j - A_k| + \left(1 - \frac{(\beta_j + \beta_k)}{2}\right) * |B_j - B_k|$$

We then assume as a first basis, that the frequency (or probability to interact) is directly the corresponding similarity i.e.  $F_{jk} = 1 - \Delta_{jk}$

When agents contact they may discuss over attitude  $A$  and  $B$ . The chances of discussing over  $A$  depend on the relative importance of  $A$ , formalised as:

$$p_{discussA} = (\beta_j + \beta_k) / 2$$

As a result from the discussion the agents may change their opinions. For this we use the formalisation as introduced by [7] and [8]. As a reminder, each individual  $j$  has got a threshold determining the latitude of acceptance  $u_i$  and a threshold determining the latitude of rejection  $T_i$  with  $T_i > U_i$ . During the interaction between agent  $j$  and agent  $k$ , the following rules are applied for agent  $j$ ’ (where  $x$  stands either for  $A$  or  $B$ ) for the central processing:

$$\begin{aligned} \text{If } |x_j - x_k| < U_j & \quad dx_j = \mu \cdot (x_k - x_j) \\ \text{If } |x_j - x_k| > T_j & \quad dx_j = \mu \cdot (x_j - x_k) \end{aligned}$$

where the parameter  $\mu$  controls for the strength of influence.

The rule describing peripheral processing is for agent  $j$ :

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<sup>1</sup> And symmetrically to agent  $k$

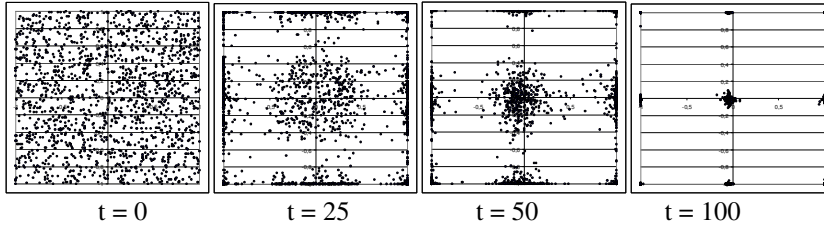
$$\begin{aligned} \text{If } |A_j - A_k| < U_j & \quad dA_j = \mu \cdot (A_k - A_j) \text{ and } dB_j = \mu \cdot (B_k - B_j) \\ \text{If } |A_j - A_k| > T_j & \quad dA_j = \mu \cdot (A_k - A_j) \text{ and } dB_j = \mu \cdot (B_j - B_k) \end{aligned}$$

Here people sharing close positions on a relevant opinion dimension are likely to become more similar (assimilation effects), whereas people really differing on a dimension might become more dissimilar (contrast effects). Also a change on the dimension being discussed (central processing) can result in a change in a similar style (assimilation or contrast) on the other dimension (peripheral processing), as explained in [10].

### 3: Experiments

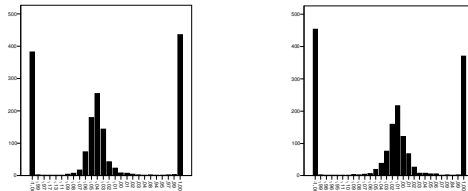
In a first experiment we replicated the experiments as performed in [8]. The strength of influence,  $\mu$  was set equal (0.2), the link density was set at 0.05, and 1600 agents were instantiated. The first condition we tested was with an acceptance threshold  $U$  of 0.5, and a rejectance threshold  $T$  of 1.5, which in the original experiment [7] with local interactions led to most agents having an extreme position on both dimensions, where the correlation between  $A$  and  $B$  was low on average. In the following figures we see for  $t = 0, 25, 50$  and  $100$  the development of the attitude position on  $A$  (x-axis, horizontal) and attitude  $B$  (y-axis vertical), both attitudes ranging from -1 to 1.

Figure 1: Development of attitude positions for  $U=0.5$ , and  $T=1.5$



As can be observed, most agents cluster in a mid position, and a limited number takes an extreme position. The following Figure 2 displays the frequencies for the position on the x-axis (left) and y-axis (right)

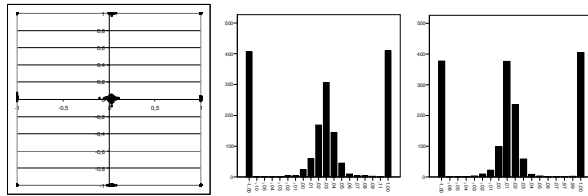
Figure 2: Frequencies of attitude positions for  $U=0.5$ , and  $T=1.5$  for x-axis (left) and y-axis (right).



As the chance of taking a mid position is large, the number of agents having an extreme position on both dimensions is relative small, which can be seen in the moderate amount of observations in the corners. Hence, if agents prefer to have contact with more similar agents, we observe under condition of small latitude of acceptance and high latitude of non-commitment basically a lot of convergence in the middle, and a limited number of extreme attitudes.

If the level of acceptance  $U$  is increased to 1.0, we observe a stronger clustering in the center, as can be seen in Figure 3.

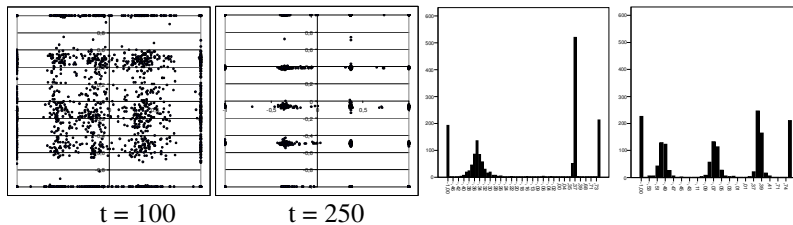
Figure 3: Development of attitude positions for  $U=1.0$ , and  $T=1.5$  for both axis, and frequencies on x-axis (middle) and y-axis (right) on  $t = 100$



Here we observe 3 clusters on both dimensions, resulting in a total of 9 clusters, the middle one being the largest.

If the acceptance and rejectance rate are set wide apart, as presented in Figure 4, we observe the emergence of a larger number of clusters (mind the different time-scale).

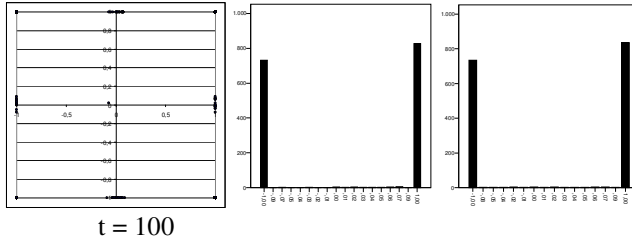
Figure 4: Development of attitude positions for  $U=0.25$ , and  $T=1.75$  for both axis, and frequencies on x-axis (middle) and y-axis (right) on  $t = 250$



This experiment reveals that on the x-axis 4 clusters emerged, one having a broad distribution. In time this will converge to a single point as well. On the y-axis we observe the emergence of 5 clusters. In total 20 clusters emerged.

In the situation where the latitude of acceptance and rejectance are very close, as presented in Figure 5, we observe a strong polarisation in the attitudes.

Figure 5: Development of attitude positions for  $U=0.9$ , and  $T=1,1$  for both axis, and frequencies on x-axis (middle) and y-axis (right) on  $t = 100$ .



Here we observe that virtually all agents are clustered in the four corners, and 4 clusters having extreme opinions emerge.

Compared with previous experiments, the FreqNet formalisation shows that due to the preferential attachment<sup>2</sup> principles less polarization occurs than in a situation with random contacts. This is due to the fact that agents interact more frequently with similar other agents, and hence they are less likely to interact with very different other agents and reject their attitudes. This suggests that preferential attachment based on attitude similarity may result in pluriformity and networks of relative similar agents. Only if the acceptance and rejectance levels get close, a strong polarization effect emerges, just as in [7] on a single attitude dimension, resulting of networks of very similar agents. Mind however that even in these conditions it is possible that agents are linked that differ e.g. extremely on  $B$  in case they both find  $B$  unimportant.

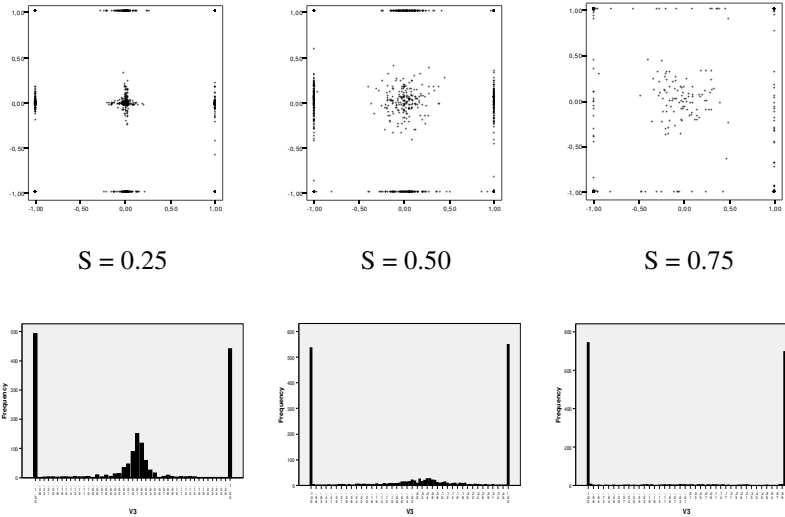
### *Experiments with peripheral processing*

In the next series of experiments, we replicated the previous experiments including peripheral processing. Here, the agents process centrally on a first dimension according to the probability corresponding to the average of the importance of this dimension for them. On the second dimension, they either engage in central processing or peripheral processing, dependant on a threshold level for peripheral processing  $S_j$ . Peripheral processing here implies that the opinion shift made on the other dimension is copied to the other dimension, regardless of the initial positions.

Below we present the replications for three thresholds for peripheral processing:  $S = 0.25, 0.50$  and  $0.75$ , the last condition implicating more peripheral processing on the second dimension. First the condition of  $0.50$  acceptance ( $U$ ) and  $1.50$  rejectance ( $T$ ) is presented.

*Figure 6: Attitude positions for  $U=0.5$  and  $T=1.5$  for  $S=0.25, 0.50$  and  $0.75$  on  $t = 100$ .*

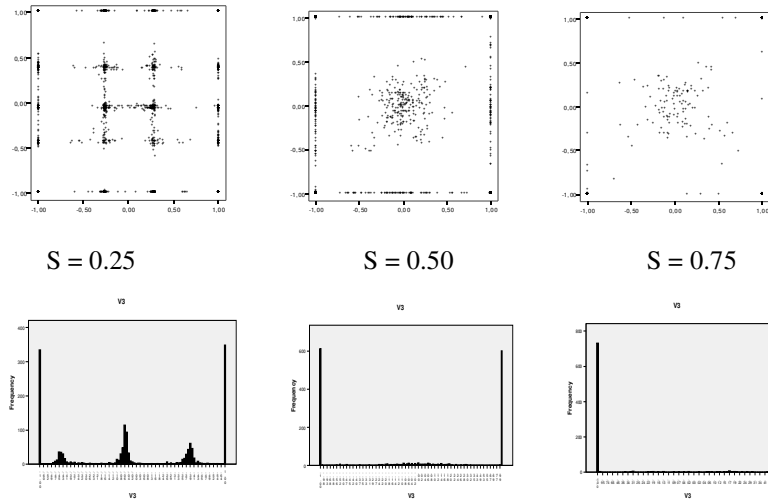
<sup>2</sup> Not in the restrictive definition proposed by the preferential attachment model of Barabasi.



As can be observed, an increase of peripheral processing causes the attitudes to polarize more on both dimensions. This is in contrast to [8], where peripheral processing reduced polarization. However, in [8] the agents centrally processed on attitude  $A$ , and peripherally on attitude  $B$ , whereas in the FreqNet formalisation agents have a chance of engaging in peripheral processing on either  $A$  or  $B$  depending on the importance of the dimension. Apparently this preferential attachment on the basis of attitude similarity causes peripheral processing to work in an opposite direction than in random contacts. We expect that due to random contacts more opposite forces are experienced, and hence a mid-position is more stable, whereas in preferential attachment the attraction force will dominate, and hence peripheral processing will only support the clustering of similar agents who become more similar in the process.

In Figure 7 we present the results for 0.25 acceptance and 1.75 rejectance, this for  $t = 250$ , as in this condition it takes more time for a relative stable solution to emerge.

Figure 7: Attitude positions for  $U=0.25$  and  $T=1.75$  for  $S=0.25, 0.50$  and  $0.75$  on  $t = 250$ .



Also in this condition we observe that an increase in peripheral processing leads towards polarization. The more peripheral processing, the smaller the chance of having a stable pluralistic distribution of attitudes.

#### 4: Conclusions and discussion

In this short paper we made some first steps in exploring the consequences of a preferential attachment process based on attitude positions, on the dynamics in these attitudes. First results indicate that if people link on the basis of similarity on attitudes, the polarization tendency becomes stronger. Also we observe that more peripheral processing results in more polarization. However, we have not yet studied the shape of networks that emerged, whereas it can be expected that expressing networks in terms of which agents interacted during the last  $n$  time-steps would reveal how social networks may evolve as a function of attitude change. Including heterogeneity in agents is also expected to reveal how agent traits, such as susceptibility to change, are related to their position in networks, e.g., centrality. A next research question we plan to address using this model is the emergence and stability of groups. Under certain conditions we observe that agents sharing an extreme position with other agents may move towards the opposite camp, and in the process they change their network in terms of frequency of interaction. Moreover, under certain conditions, i.e. contrast effects are stronger than attraction effects, the model does not converge to a stable situation, but the attitudes of some agents, and their networks keep changing. In the future many questions may be addressed using this FreqNet approach, e.g. the effects of heterogeneity in number of contacts, firmness of beliefs, normative and informative influences and the like. However, the first step we have scheduled is the identification of network shapes, and relating the dynamics of these shapes to interactions on attitude dimensions. As such our main challenge is to use FreqNet as to explore how different network configurations may emerge as a function of individual traits influencing processes of social interactions.

## References

- [1] Goldenberg J., Libai B., Solomon S., Neam J. and Stauffer D. (2000), "Marketing Percolation", *Physica A*, vol.284, pp.335-347.
- [2] Milgram, S. (1967), "The Small World Problem", *Psychology Today*, May 1967, pp.60-67.
- [3] Watts, D.J. (1999), *Small Worlds: The Dynamics of Networks Between Order and Randomness*, Princeton University Press, Princeton.
- [4] Barabasi, A.-L. and Albert, R. (1999), "Emergence of Scaling in Random Networks", *Science*, vol.286, pp.509-512.
- [5] Amaral L.A.N, Scala A., Barthemely M. and Stanley H.E. (2000), "Classes of small-world networks", *Proceedings of the National Academy of Sciences USA*, vol.97, pp.11149-11152.
- [6] Festinger L., (1954), "A theory of social comparison processes", *Human Relations*, vol.7, pp.117-140.
- [7] Jager W. and Amblard F. (2004), "Uniformity, Bipolarization and Pluriformity Captured as Generic Stylized Behavior with an Agent-Based Simulation Model of Attitude Change", *Computational & Mathematical Organization Theory*, vol.10 (4), pp.295-303.
- [8] Jager W. and Amblard F. (2007), "Guess you're right on this one too: Central and peripheral processing in attitude changes in large populations", *Advancing Social Simulation*, S. Takahashi, D. Sallach, J. Rouchier (eds.), Springer, p.249-260.
- [9] Dorogovtsev S.N. and Mendes J.F.F. (2000), "Evolution of reference networks with aging", *Physical Review E*, vol.62, pp.1842.
- [10] Petty, R.E. and Cacioppo, J.T., (1986). *Communication and persuasion: central and peripheral routes to attitude change*. New York: Springer Verlag.